Financing Transplant Costs of the Poor: A Dynamic Model of Global Kidney Exchange*

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Abstract

In some developing nations, many end-stage renal disease patients die because health insurance does not cover kidney transplantation. We analyze two proposals for extending kidney exchange to include such patients. In the first, foreign patient-donor pairs exchange with an American patient-donor pair, for free. A dynamic model shows this can be self-financing even when the average dialysis cost of American patients declines below the costs of the surgeries (as waiting times for transplant are shortened by increased availability). We also consider including foreign pairs associated with a bridge donor who could donate a kidney to someone waiting for a deceased donors.

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1 Introduction

Transplantation is the treatment of choice for kidney failure, but there are many barriers that prevent transplantation around the world. These barriers are different in economically developed countries than in developing countries, but transplantation is woefully undersupplied relative to the prevalence of kidney failure in both rich and poor nations. In much of the developing world, transplantation or other treatment such as dialysis is largely unavailable due to shortages of medical facilities and finances, and kidney failure is a death sentence. In the developed world many people must struggle with dialysis while enduring long waits for transplantation, with many thousands of patients dying each year while waiting, due to the shortage of transplantable organs [Coresh and Jafar 2015, Liyanage et al. 2015].

This paper considers how these two problems—lack of access to transplantation or dialysis, and the shortage of transplantable organs—can each help alleviate the other, by extending the growing practice of kidney exchange globally. Kidney exchange can increase the number of transplants available from willing living donors who are otherwise unable to achieve their wish to donate a kidney. By inviting foreign patients and their donors to participate in American kidney exchange, both American and foreign patient-donor pairs will have increased opportunities to receive a transplant. And (as we will see) the resulting savings to the American health care system from moving an American patient off dialysis will be sufficient in the steady state to finance the cost of transplantation and post-surgical care for the foreign patients and donors, even after such a program increases the availability of transplants to the point that it substantially reduces dialysis costs for American patients. (We use American kidney exchange as an example, but the proposals outlined here would work similarly in other developed nations with kidney exchange involving hard to match pairs and expensive dialysis.)

We analyze two proposals for extending kidney exchange to include patients in countries in which transplantation is unavailable to them. First, we analyze a recently proposed concept, Global Kidney Exchange (GKE), in which a U.S. health authority invites patients with financial restrictions who have willing donors to come to the United States to exchange their donor's kidney with an immunologically incompatible American patient-donor pair and to receive a transplant utilizing the incompatible American donor's kidney for free ([Rees et al. 2017a] reports the first completed GKE transplant). The ability of such a program to benefit foreign patient-donor pairs would be limited only by the number of American patient-donor pairs with whom they could be matched, which is a pool of pairs in the single digit thousands. We also introduce a new proposal, the Non-Directed Donor proposal, which would relax this limit, by including foreign pairs that could be associated with a bridge donor in a non-directed donor chain who could donate a kidney to a patient on the U.S. waiting list for deceased donors.[1] The ability of such a program to benefit foreign patient-donor pairs associated with non-directed donor chains would be limited only by the total number of transplant-eligible American patients with ESRD, which includes at least the hundred thousand currently on the deceased donor waiting list.

1.1 Background and a static analysis

In the United States there are presently about 100,000 people on the waiting list for a deceased donor kidney, but only around 14,000 such kidneys were available in 2017. Approximately 6,000 additional transplants were made with kidneys donated by living donors. Live donation is possible because a healthy person has two kidneys and can remain healthy with just one. But not everyone who is healthy enough to donate a kidney can donate to their intended recipient, since a kidney must be compatible with the recipient to be successfully transplanted. Due to the shortage of transplant opportunities, about 4,000 people on the U.S. waiting list died in 2017, and almost 5,000 more were taken off the waiting list when they became too sick to transplant. [2]

To help alleviate the shortage of transplantable organs, kidney exchange, which allows patient-donor pairs to exchange kidneys (so that each patient receives a compatible kidney), has become a standard mode of transplantation in the last decade. (See the history and references in [Wallis et al. 2011], and e.g. [Roth et al. 2004, Ashlagi and Roth 2014, Roth 2015, Fumo et al. 2015]) Since 2013, well over 10% of the live kidney donor transplants in the U.S. each year were accomplished through exchange.[3] However because some patients are "highly sensitized", for whom it is hard to find a compatible donor, a substantial fraction of incompatible patient-donor pairs cannot be transplanted through simple exchanges. The majority of transplants that are found for those hard-to-match pairs are organized in chains of transplants that begin with a non-directed donor, i.e. a donor willing to give a kidney without receiving one in exchange (see [Rees et al. 2009, Ashlagi et al. 2014, Anderson et al. 2015]). Through 2017 there have been over 2,000 anonymous non-directed donations in the U.S., as well as almost 25,000 donations from unrelated donors who were not anonymous at

[1] That is, a foreign non-directed donor could start a kidney exchange chain in the foreign country, with the last donor in the chain donating to an American.

[2] See https://optn.transplant.hrsa.gov/data/view-data-reports/national-data/, accessed 1 May 2018.

[3] http://optn.transplant.hrsa.gov/converge/latestData/rptData.asp (see kidney transplants by donor relation).

the time of donation.[4]

In the U.S., kidney failure—more precisely, end stage renal disease (ESRD)—accounts for about 6% of the Medicare budget, and together with the costs borne by private payers has annual costs of around $50 billion [USRDS 2017]. Most of this is the cost of dialysis, which costs Medicare about $90,000 per year per patient, compared to kidney transplantation, which costs about $90,000, followed by about $15,000 per year of immunosuppressive drugs [USRDS 2017, Irwin et al. 2015]. Thus a transplant to an American plus a transplant for a foreign pair saves Medicare about $120,000 in the first five years.[5]

Patients who have comprehensive medical insurance, typically through an employer, become eligible for primary Medicare coverage of dialysis only after 33 months of treatment; beforehand the private insurer foots most of the bill, at a cost typically over twice the rate at which Medicare is billed [6][Hippen 2015]. Consequently, savings from transplanting an American patient also accrue to private insurers.

On the other hand, it is estimated that in 2010, only a quarter to a half of the world?s patients in need of renal replacement therapies received it, 92.8% of whom resided in high-income or high middle income countries [Liyanage et al. 2015]. Thus the shortage of kidneys in the U.S. and elsewhere in the developed world, together with the unavailability of transplantation to many people in the developing world, presents an opportunity for mutually beneficial exchange, by extending kidney exchange beyond domestic patient-donor pairs.

In the appendix we set up a simple model that captures this static cost-benefit analysis. To implement GKE at scale, however, a more careful analysis is needed as the above back of the envelope analysis does not account for the dynamic aspects of the problem. The important aspects captured in our dynamic cost-benefit analysis, are: (i) Domestic patients who have not found a match have a higher average dialysis cost than the average dialysis cost of all domestic patients, (ii) There is an option-value of keeping domestic pairs in the pool, and (iii) As GKE starts being implemented at scale, the average dialysis and surgery costs (per patient) change.

[4] https://optn.transplant.hrsa.gov/data/view-data-reports/national-data/ accessed May 2, 2018. (Non-Biol,unrel: Anonymous Donation= 2,074 through 2017, while Non-Biological, Other Unrelated Directed = 24,800) Some of the donations that were directed and not anonymous at the time of donation will have originated as non-directed donations, but been recorded as directed donations after a match was made at a transplant center.

[5] In case of a kidney graft failure, [Held et al. 2015] attribute an additional cost of $80,000, which reduces the savings to $40,000. This is a conservative financial estimate and does not count the benefits of improved health to patients - see e.g. [Held et al. 2015]

[6] Medicare Coverage of Kidney Dialysis & Kidney Transplant Services http://www.medicare.gov/Pubs/pdf/10128.pdfpage14

These considerations change the simple static analysis. First, the static analysis could underestimate the savings of GKE, as it understates the average dialysis cost of the domestic patient involved in the exchange. Second, the static analysis could overestimate the savings of GKE: by allowing an exchange between a domestic and an international pair, the planner eliminates the potential option of a future exchange between domestic pairs. In other words, the domestic pool becomes thinner in the presence of a steady arrival of international pairs , which would reduce the number of future domestic exchanges. Our dynamic steady-state analysis captures this option value of keeping the domestic pairs in the pool. And third, the estimates used in the static analysis change when GKE is implemented. As the arrival rate of international pairs increases, the expected dialysis cost for a given domestic patient decreases, while the expected cost paid for her surgery increases.

The decrease in the expected dialysis cost creates a force in favor of bringing fewer international pairs, while the increase in the expected surgery cost creates a force in favor of bringing more international pairs: in a transplant involving an international and a domestic patient, the expected *additional* cost paid for the domestic patient also depends on the chance that she otherwise gets a transplant in the future. The higher the chance, the smaller the expected additional surgery cost paid for her, which can make the planner more willing to arrange an earlier exchange for her with an international pair to avoid her dialysis costs.

In the analysis of the dynamic model, we show that the forces that work in favor of bringing more international pairs dominate the forces in favor of bringing fewer even when the average dialysis cost for a domestic patient is *lower* than the surgery costs. In other words, GKE can reduce the total healthcare costs even to the point when the average cost of dialysis drops below the surgery costs.

It is worth pausing for a moment to consider the uses of stylized models that are quite different from the detailed models that will be needed to actually implement exchange in particular clinical populations [Ashlagi et al. 2011b;a]. Detailed models of clinical conditions allow optimization and simulation involving the particular patients and donors who are available, but make it difficult to draw robust general conclusions, because of the important heterogeneities among patients, the complicated exchanges needed to maximize transplants, etc. In contrast, simple stochastic models allow us to see how costs change as the number of international pairs is increased, and waiting times of domestic patients decrease. To achieve this simplicity, we concentrate here on two-way exchanges among uniformly highly sensitized patients. The assumption that patients are uniformly highly sensitized (and hence are part of sparse compatibility graphs) is justified because it is the highly sensitized domestic patients who would be the most likely candidates for exchange with foreign pairs. And modeling exchanges as two-way exchanges is a conservative assumption, since we consider how even exchange among just two patient-donor pairs could be financed in the steady state by the savings associated with taking a single domestic patient off dialysis.

1.2 Two Proposals

The first proposal referred to in the introduction, global kidney exchange (GKE), would invite foreign patient-donor pairs to come to the U.S. to receive a kidney through an exchange with a domestic pair, with the costs to be paid from the savings to Medicare or private insurance from transplanting (and thus removing from dialysis) an American patient. [Krawiec and Rees 2014] introduce this proposal under the name of "reverse transplant tourism" and argue that it is legal under the existing American statutes governing kidney transplantation. In January 2015, the Alliance for Paired Donation organized a first such surgery for a Filipino husband and wife patient and donor. The surgery and post-surgical care for both patient and donor were funded by $150,000 of philanthropy (see Rees et al. 2017).[7]

The new proposal, the non-directed donor proposal (NDD), would include foreign pairs that could be associated with a non-directed donor chain, and whose paired donor would donate a kidney to a patient on the U.S. waiting list for deceased donors. The patient in such a foreign pair could receive a kidney as part of the non-directed donor chain in his or her home country, with the costs again to be paid by the savings to Medicare or private insurance from transplanting an American patient. (In practice several foreign transplants could be funded by starting a longer chain of transplants with a non-directed donor in the foreign location, with a bridge donor travelling to the U.S. to continue the chain or donate to a patient on the American deceased donor waiting list. But to be conservative, we consider here the simplest case in which only one foreign pair and one domestic recipient are involved in a short chain in which the non-directed donor gives to the foreign patient and the bridge donor gives to an American patient.)

Proposals like these raise questions both about practicality, and about ethics. This paper is thus concerned both with how ethical considerations can be addressed with practical

[7] The Filipino pair received a kidney from a non-directed American donor in Georgia, and in turn donated to an American patient in Minnesota, whose donor continued the chain by donating to a patient in Seattle. Eventually the chain expanded to include 11 patient-donor pairs. So an additional patient-donor pair can sometimes facilitate a chain that allows more than just one American patient to be transplanted. In the present paper we will consider how the system can be made self-financing (even) under the conservative assumption that each international pair would facilitate only one American transplant.

solutions, and how ethical considerations may constrain the forms that practical solutions can take. We will model the practical issues, and discuss how the ethical issues raised by these proposals are similar to and different from ethical issues that have been encountered in establishing kidney exchange in the last decade.

The practicality issues that this paper addresses concern the financial sustainability of the GKE and NDD proposals in the long term.

The novel ethical concerns raised by kidney exchange chiefly involve the treatment, care and rights of living donors. Some of these concerns have already arisen, and been addressed, in the development of kidney exchange within the United States and in other developed countries. Additional ethical concerns will inevitably arise in any proposal that involves organ donations in developing countries, particularly in light of the fact that the purchase and sale of kidneys for transplant is almost universally illegal.[8] One focus of the present paper will be to consider the nature of potential objections to the present proposals, what potential harms might accompany such proposals, how they might be addressed, and what kind of monitoring of such programs might therefore be called for.

We analyze these two proposals from the perspective of two different planners: a private profit-maximizing insurance corporation and the U.S. State Department. The private profit-maximizing insurance corporation seeks to add overseas patients and donors so as to minimize its total costs. The State Department's objective is to provide transplants as a form of foreign aid, and thus to maximize the number of international patients subject to the constraint that at least as many domestic patients should be transplanted as in the status quo, and subject to the self-financing constraint that the total healthcare cost should not increase compared to the status quo.

We introduce the GKE proposal and its cost-benefit analysis in Section 2. The NDD proposal and its cost-benefit analysis are introduced in Section 3. We discuss the ethical concerns that arise with these proposals in Section 4, and the design of financial flows in Section 5. Section 6 is the conclusion. All proofs are presented in the appendices.

[8] That is, cash payment for kidneys is almost universally enshrined in laws as a repugnant transaction in the sense of being a transaction that some people would like to engage in but others find objectionable without the presence of the usual sorts of direct negative externalities (see [Roth 2007]). For more on repugnance specifically to kidney sales, or less direct compensation to donors, see [Leider and Roth 2010] and [Niederle and Roth 2014].

2 The Global Kidney Exchange Proposal

Global Kidney Exchange would invite foreign patient-donor pairs to come to the U.S. to receive a kidney through an exchange with a domestic pair, with all the associated costs covered by an American institution.

To fix ideas, suppose A is a domestic patient and B is her brother who is willing to donate a kidney to her, but they are immunologically incompatible. In addition, suppose C is an international patient and D is her brother who is willing to donate a kidney to her, but they are financially constrained and cannot afford transplant or maintenance dialysis therapy costs. Now suppose B is biologically compatible with C, and D is biologically compatible with A. Under the GKE proposal, the C-D pair can have an exchange with the A-B pair.

The domestic planner (the State Department or a profit-maximizing private insurance company) pays *all* of the surgery and maintenance therapy costs for both transplants. We will show that the savings in the dialysis costs of domestic patients can cover the costs of additional transplants and maintenance therapy. Thus, the GKE proposal can be self-financing.

It is worth noting that ESRD patients are either *highly sensitized* (*i.e.*, it is hard to find a match for them) or *low sensitized* (i.e. it is easy to find a match for them). The fraction of highly sensitized patients is higher in developed countries, because those who have already had a kidney transplant are more likely to be highly sensitized. These patients need to wait for a relatively long time before, if at all, getting matched and so their dialysis costs are higher than the average. Under the GKE proposal, domestic pairs with highly sensitized patients are likely to be matched to international pairs with low sensitized patients.[9]

2.1 Cost-Benefit Analysis of the GKE Proposal

In Section 1, we saw a simple accounting argument based on the dialysis and transplant costs in the United States that suggested the GKE proposal can reduce the healthcare costs of the average domestic patient in the first five years.

Here we propose and analyze a stochastic model for GKE.[10] Our steady-state cost-benefit

[9] Identifying a match between an international pair and a highly-sensitized American pair may involve additional costs but our argument will be unaffected so long as these costs do not bring the surgery cost much above the average dialysis cost, in the sense of Theorems 2.2 and C.1.

[10] Our baseline model generalizes the model introduced in [Akbarpour et al. 2017] in multiple ways: We have two types of agents while in [Akbarpour et al. 2017] agents are *ex ante* homogeneous. We also have a continuum mass of agents, as opposed to a discrete state space. This is for expositional simplicity. We study a discrete version of the model through simulations (in appendix F) and analytically (in the online

analysis shows that GKE remains self-financing even when the average dialysis cost drops *below* the surgery cost (which, clearly, the accounting argument is incapable of capturing). Furthermore, we pin down the limits of GKE by characterizing an economically meaningful necessary and sufficient condition under which bringing (marginally) more international patients would reduce the total healthcare costs.

2.2 The Dynamic GKE Model

Arrivals and departures. There are two types of patient-donor pairs in our model: *domestic* pairs and *international* pairs. Domestic patient-donor pairs have an arrival flow with rate m; therefore, a measure $m\,dt$ of such pairs arrive in a time interval of length dt. International pairs have an arrival flow with rate λm, where $\lambda \geq 0$ is a policy decision. Pairs remain in the pool until they are matched or they depart from the pool. We also assume that international pairs cannot be matched together, but they can be matched to domestic pairs. Domestic pairs can be matched together, or to international pairs, according to the matching technology we formalize next.

Every pair (either domestic or international) exits the market (unmatched) at rate η which, without loss of generality, we normalize to 1. We call the measure of pairs waiting in the market the *size of the pool*, and use x, y to denote the sizes of domestic and international pools, respectively.

Matching technology. To define our matching technology, we start by assuming $\lambda = 0$. Suppose the domestic pool has size x at some time t and an infinitesimal measure mdt of pairs arrive to the pool in the time interval $[t, t + dt]$. Then, a measure $\mu(x) \cdot mdt$ of the arriving pairs will be matched to an equal measure of the pairs waiting in the pool, where $\mu : \mathbb{R}_{++} \to [0, 1]$ is the *matching function.* $\mu(x)$ can be interpreted as the chance that a pair who has just arrived can find a compatible pair in a pool of size x. The choice of the matching function plays an important role in the dynamics of the model. For matching functions in a steady state system, the pool size Z is determined from the balance equation $m(1 - \mu(Z)) = Z + m\mu(Z)$.

Recall that at this point we are assuming there are no international pairs. Then, the left-hand side of the equation denotes the arrival rate to the pool since $(1 - \mu(Z))$ fraction of newly arrived pairs are not matched upon arrival. The right-hand side of the equation

addendum of the paper).

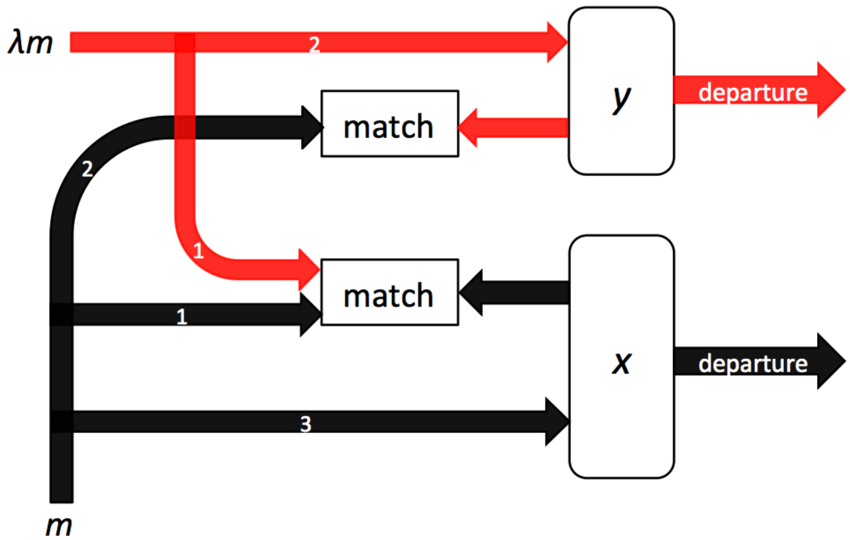


Figure 1: Dynamics of the global kidney exchange model. Domestic pairs arrive with rate m, and the planner first tries to match them to other domestic pairs, then to international pairs, and if there is no match, they will enter the pool of domestic pairs. International pairs arrive with rate λm and the planner tries to match them to domestic pairs and if they are not matched, they will enter the pool of international pairs. All pairs depart at some stochastic rate.

denotes the departure rates from pool, since $m\mu(Z)$ of pairs are matched to newly arrived pairs and each waiting pair departs at rate 1.

Define $\overline{\mu}(x) = 1 - \mu(x)$ as the chance that an arriving pair cannot find a compatible match in a pool of size x. Assume the matching function satisfies the property

$$\overline{\mu}(x + y) = \overline{\mu}(x) \cdot \overline{\mu}(y),$$

which could be interpreted as an "independence assumption": the chance that a pair does not have a compatible match in the union of two disjoint pools with sizes x, y is equal to the product of the chances that the pair does not have a compatible match in either of the pools. Then, we must have

$$\log \overline{\mu}(x + y) = \log \overline{\mu}(x) + \log \overline{\mu}(y).$$

This means that $\log \overline{\mu}$ must be linear in its argument. So, we must have $\log \overline{\mu}(x) = \alpha x$. The condition $\overline{\mu}(x) \leq 1$ implies $\alpha \geq 0$. We therefore choose the matching function $\mu(x) = 1-e^{-\gamma x}$, where $\gamma > 0$, the match-rate parameter, is a constant.[11]

In Section 2.3, we micro-found this matching function and observe that our model approximates its natural discrete counterpart quite well. The discrete model assumes poisson arrival rates for patient-donor pairs whose compatibility to each other is probabilistic (and therefore does not use a matching function).

Matching policy. The matching policy of the planner is as follows: Whenever new domestic pairs arrive, the planner first attempts to match them to domestic pairs who are waiting in the pool, and then attempts to match them to international pairs. If no matches are found, they will enter the domestic pool and wait. Whenever international pairs arrive, the planner attempts to match them to domestic pairs, and if they do not find a match, they enter the international pool and wait.

Steady state. Let x, y be the steady state sizes of the domestic and international pools, respectively. Then, the balance equations under steady state are:

$$m\bar{\mu}(x)\bar{\mu}(y) = m(1+\lambda)\mu(x) + x \tag{2.1}$$

$$\lambda m\bar{\mu}(x) = m\bar{\mu}(x)\mu(y) + y \tag{2.2}$$

The left hand side of each of the equations is the rate at which new domestic and international pairs are unmatched and hence added to their respective pools, and the right hand side is the rate at which those pools are diminished as agents are matched or otherwise depart (recall that the departure rate of incumbents is normalized to 1). The following proposition establishes some basic properties of the solution to the system of equations defined by (2.1) and (2.2). (All missing proofs are in the appendix)

Proposition 2.1. *Fix $m > 0$. Then, for any $\lambda \in [0, 1)$, the system of equations given by (2.1) and (2.2) has a unique solution, denoted by $x(\lambda), y(\lambda)$. Furthermore, the functions $x(\lambda), y(\lambda) : [0, 1) \to \mathbb{R}$ are continuously differentiable on $[0, 1)$.*

Cost functions. Our goal is to study the healthcare costs of the system. For that, let s be the *surgery cost* of a unit mass of pairs. Also, let d be the per period (unit of time)

[11] For simplicity we assume that the matching function is the same for the domestic and international pools, but the fact that international patients are typically easier to match makes this a conservative assumption.

dialysis cost of a unit mass of pairs. Let $w(\lambda)$ be the *aggregate waiting time* of domestic pairs per period. Observe that $w(\lambda) = x(\lambda)$. The *total dialysis cost* per period is defined as $C_d(\lambda) = d \cdot w(\lambda)$.

Let $\mathcal{M}(\lambda)$ denote the per period measure of agents (domestic or international) who get matched, for any given policy choice λ. The *total surgery cost* per period is defined as $C_s(\lambda) = s \cdot \mathcal{M}(\lambda)$. We define the *total healthcare cost* to be $\mathbf{C}(\lambda) = C_d(\lambda) + C_s(\lambda)$.

One last definition is required to state the results: Let $\Theta(\lambda) = \frac{C_d(\lambda)}{m}/s$, which measures the relative ratio of the average dialysis cost per patient to the surgery cost per patient. In other words, $\Theta(\lambda)$ is the relative ratio of average dialysis cost for a unit mass of patients to the surgery cost for a unit mass of patients. Let $\theta = \Theta(0)$ be the status quo value of this function.

2.3 Results

We show that the GKE proposal as modeled here is *self-financing*, i.e. it does not increase the total (domestic) healthcare cost. To do this, we establish a stronger result: GKE could in fact decrease the total healthcare costs, which also includes the surgery costs of domestic patients. Therefore, the planner could benefit from the GKE proposal even when she does *not* assign any value to the additional saved lives, or to the disutility of patients who are under dialysis. Incorporating these additional values, obviously, only supports our main claim. Furthermore, GKE remains self financing even when the average dialysis cost drops below the cost of a surgery.

Theorem 2.2. *For any match-rate parameter $\gamma > 0$, there exists a constant m_γ such that for all $m > m_\gamma$, $\mathbf{C}'(0) < 0$ if $\theta > \ln(2)$. Furthermore, the constant $\ln(2)$ is tight: if $\theta < ln(2)$, then there exists m_γ such that for all $m > m_\gamma$, $\mathbf{C}'(0) > 0$.*

Theorem 2.2 shows that when the arrival rate of domestic pairs m is not too small, there exists some constant $\lambda_m > 0$ such that GKE(λ) is self-financing for all $\lambda \in [0, \lambda_m]$, if $\theta > \ln(2) \approx 0.69$ (i.e. even for some $\theta < 1$).[12]

Theorem 2.3. *Fix $\gamma > 0$. Then, for any constant $\epsilon > 0$, there exists a constant m_ϵ such that for all $m > m_\epsilon$ and any $\lambda \in [0, 1]$, $\mathbf{C}'(\lambda) < 0$ holds if $\Theta(\lambda) > \ln(2) + \epsilon$.*

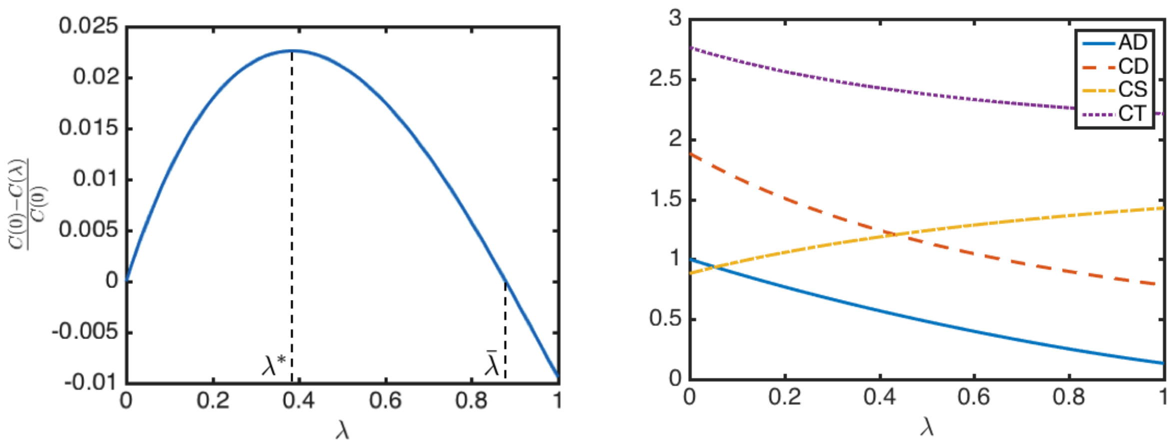
Theorem 2.3 shows that if a marginal increase in λ at $\lambda = 0$ is self-financing, then in fact we can increase λ so long as the average dialysis cost per person is more than a fraction

[12]The appearance of a natural logarithm in this result is related to the exponential form of the matching function and the fact that we allow the arrival rate m of domestic pairs to be large.

ln(2) of the surgery cost. So, even when the average dialysis cost per person *falls below* the surgery cost (per person), total health care costs will decrease when λ is increased, so long as $\Theta(\lambda) > \ln(2)$.

We next provide some intuition through an example, and discuss why increasing the arrival rate of international pairs may still reduce the healthcare costs when the average dialysis cost drops below the surgery cost. After that we provide a brief proof sketch. We also compare our stead-state analysis to a simple static model in Section A.

Example The percentage of reduction in total healthcare costs in terms of λ is plotted in Figure 2(a) for an instance of the problem with $m = 10, \gamma = 1, \theta = 1$. At $\lambda^* \approx 0.4$, $\mathbf{C}'(\lambda^*) = 0$. GKE($\lambda$), however, continues to be self-financing even for higher values of λ up to when $\lambda = \overline{\lambda}$, where $\overline{\lambda} \approx 0.87$. Observe that $\Theta(\lambda^*) \approx 0.6$, which is a smaller number than $\ln(2) \approx 0.69$, the sufficient bound given by Theorem 2.3.



(a) The value of $\frac{\mathbf{C}(0)-\mathbf{C}(\lambda)}{\mathbf{C}(0)}$ is plotted.

(b) Statistics related to average dialysis and surgery costs of domestic patients (normalized by cost of a surgery).

Figure 2: Reduction of healthcare costs in λ.

The parameters λ^* and $\overline{\lambda}$ have important interpretations: they are the choice variables of two different planners, namely, the private insurer and the domestic social planner (e.g. the State department). The private insurer seeks to add international pairs to minimize its total costs.[13] Therefore, this planner chooses $\lambda = \lambda^*$, the (unique) maximizer of $\mathbf{C}(0) - \mathbf{C}(\lambda)$. On the other hand, the State department's goal is to maximize the number of international

[13]In particular, we anticipate a steady state in which GKE will not immediately serve all domestic patients, who will therefore continue to have non-zero waiting time on dialysis.

patients subject to the constraint that at least as many domestic patients should be transplanted as in the status quo, and subject to the self-financing constraint. Therefore, the State department's choice will be $\lambda = \overline{\lambda}$.

Figure 2(b) plots several statistics related to average dialysis and surgery costs of domestic patients. The ratio of the average dialysis cost to the cost of a surgery is given by the curve AD; by definition this curve coincides with $\Theta(\lambda)$. We normalize the cost of a surgery to 1, and therefore AD could be considered as the average dialysis cost. Even though the average dialysis cost is always at most 1, $\mathbf{C}(\lambda) < \mathbf{C}(0)$ holds for all $\lambda < \overline{\lambda}$.

There are several forces involved, some strengthening and some weakening this effect. First, international patients are matched only to the domestic patients waiting in the pool, who have a higher average dialysis cost than the unconditional average dialysis cost. CD is the average dialysis cost of a patient conditioned on entering the domestic pool. (See Figure 2(b).) While CD is larger than AD for all λ, it always remains below 2, the normalized cost for two surgeries. That is, even when the cost of two surgeries (one for the international and one for the domestic patient) is higher than CD, $\mathbf{C}'(\lambda) < 0$ could still hold. The reason is the (average) surgery costs of the domestic patients waiting in the pool: the average healthcare cost of a waiting patient also includes her average surgery cost, i.e. the chance that she gets a transplant times the related surgery costs conditioned on her getting a transplant (which may or may not include the surgery cost for an international patient).

This average surgery cost is given by the curve CS. The average healthcare cost of a domestic patient waiting in the pool is just the sum of its average surgery costs (CS) and its average dialysis costs (CD); this sum is given by the curve CT. Observe that CT is always above 2, and therefore, one might expect $\mathbf{C}'(\lambda) < 0$ to always hold. This is not correct, e.g., take any $\lambda \in [\lambda^*, 1]$. The reason is the "option-value" of keeping domestic pairs in the pool: by matching a domestic pair to an international pair, we lose the option of matching the domestic pair to future domestic pairs. The (incorrect) argument that $\mathrm{CT}(\lambda) > 2$ should imply $\mathbf{C}'(\lambda) < 0$ does not account for the loss of this option-value.

Finally, observe that CT decreases more slowly than CD: the reason is that CT also includes the average surgery cost of a waiting domestic patient, which is increasing in λ. Therefore, the increasing average surgery costs of domestic patients also creates a force that works in favor of increasing λ.

Our continuum model approximates its natural discrete counterpart quite well. In the discretized model, we suppose domestic and international agents arrive according to Poisson processes with rates $n, \lambda n$, respectively. We also suppose that any two agents in the model are compatible with a probability p, independently. This assumption explicitly models the compatibility of pairs at a micro level, and therefore, there is no need for a matching function in the discrete model. All else, including the matching policy (Figure 9) and departure rate of agents, remains the same.

Figure 3 plots the percentage of reduction in the discrete model with $n = 1000$ and $p = 0.01$. Observe that the percentages given by Figure 3 match the ones given by Figure 2(a) quite well (i.e. observe that crosses and dots almost coincide in Figure 3.) This is not a coincidence: Suppose that any 100 agents in the discrete model are roughly equivalent to a unit mass of agents in the continuum model. Therefore, the arrival rate in a continuum model that approximates the discrete model should be $m = 10$. What about γ? Observe that in the discrete model, the probability that a single agent is not compatible to a group of 100 agents is $(1-p)^{100} \approx e^{-1}$. In the continuum model, the chance that an (infinitesimal) agent is compatible to a unit mass of agents is $e^{-\gamma}$, by definition. Therefore, setting $\gamma = 1$ should give a reasonable approximation. This is indeed the case, as Figure 3 suggests. In this figure we compare the estimated ratio of reduction in healthcare costs in the discrete model to the analytical solution from the continuum model. The estimated ratio in the discrete model is obtained using computer simulations (Section F).

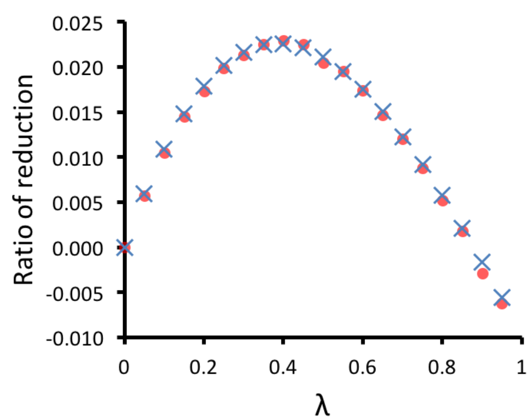


Figure 3: The red dots represent the estimated ratio of reduction in healthcare costs in the discrete model with $n = 1000, p = 0.01$, and $\theta = 1$. The estimate is computed by simulating the process for $\lambda \in \{0, 0.05, 0.1, \ldots, 0.95\}$. For any such λ, we run the stochastic process until 3×10^6 agents arrive. The blue crosses are the analytical solution to the continuum model that approximates the discrete model.

Proof sketch for Theorem 2.3 We do not directly solve the system of equations (2.1), (2.2) to prove the theorem. Rather, we compute a closed-form expression for $\mathbf{C}'(\lambda)$. To derive this closed-form expression, we take the total derivative of equations (2.1) and (2.2) with respect to λ. This will give us closed-form expressions for $x'(\lambda), y'(\lambda)$, which are in terms of $x(\lambda), y(\lambda)$. (Recall that $x(\lambda), y(\lambda)$ denote the size of the domestic and international pool when the arrival rate of international pairs is set to λm.) These closed form expressions and some algebraic calculations then reveal that $\mathbf{C}'(\lambda) > 0$ holds iff

$$\Theta(\lambda) > \frac{2\gamma x(\lambda)}{e^{\gamma x(\lambda)}} + \frac{2x(\lambda)}{m}. \tag{2.3}$$

We define $f_m(\lambda)$ to be the right-hand side of (2.3). By investigating this functional form (the right-hand side) further, we establish that (i) it is decreasing in λ, (ii) $f_m(0)$ gets close to $\ln(2)$ for sufficiently large m. The theorem is a direct consequence of these two facts.

3 Saving Many More Lives: The Non-Directed Donors Proposal

The ability of GKE to benefit foreign patient-donor pairs is limited only by the number of domestic pairs with whom international pairs could be matched. In the United States, the number of those pairs is less than $10,000$.

Our second proposal, the *Non-Directed Donors* (NDD) proposal, would relax this limit. Under this proposal, an international non-directed donor who wished to donate a kidney to a patient in his country, but could not do so because of the unavailability of transplantation financing there, can facilitate a transplant for a patient of his country by becoming an altruistic donor in a chain that would begin in their home country but continue in the sponsoring country. In the simplest, shortest chain, the nondirected donor would donate to a pair in their home country, and the donor associated with that patient would donate to a person waiting for a deceased donor in the sponsoring country. In the United States, the total number of patients who are on the waiting list for deceased donors is more than $100,000$.

The NDD proposal considered here thus works as follows. Suppose A is an international patient who needs a kidney and her brother, B, is willing to donate a kidney to her. In addition, suppose C is a non-directed donor who is willing to donate his kidney to a fellow citizen, but because of the unavailability of transplantation finances, he cannot do so. Under the NDD proposal, C donates his kidney to A, and B donates his kidney to an American patient who is on the waiting list, while all the transplants are financed by the American planner. The donation by B facilitates C's donation to A, since (as we will show) the savings in the dialysis costs by removing the American patient from the national waiting list is enough to finance all transplants in a steady state.

3.1 Cost-Benefit Analysis of the NDD Proposal

An argument similar to the accounting argument that we used to show that GKE is self-financing can show that the NDD proposal is self-financing in the short-run. A successful implementation of the NDD proposal, however, reduces the waiting time (and so, the dialysis costs) of the domestic patients; therefore, the NDD proposal might not remain self-financing.

In Appendices D and E, we set up a dynamic model of the NDD proposal and identify the sufficient and necessary conditions under which this proposal remains self-financing in the long-run. We also analyze this proposal from the perspective of the two planners. Our analysis involves two closely connected models which differ only in agents' departure process. In one model, agents depart stochastically (see D for details). In the other one, agents have a fixed departure time (see E for details). The motivation for the latter model comes from the fact that, under the current practices of the U.S. healthcare system, private insurance providers will pay only for the first 33 months of dialysis and then the patients costs are paid by Medicare.

We sketch the model and its high-level analysis below, and then we discuss the planners' choices based on the analysis.

The national waiting list for kidneys is essentially an "overloaded queue", in which the arrival rate of patients to the system is more than the current service rate, i.e. the arrival rate of deceased donors. More formally, suppose domestic patients arrive to the national waiting list at a rate m_d. In the benchmarked case, they stay in the queue for a stochastic time interval drawn from an exponential distribution of rate 1. In the fixed departure model, they stay in the market for one unit of time and then leave (i.e., they have a unit sojourn time). Let $\bar{k}$ be the arrival rate of kidneys from deceased donors in the status quo. The queue is overloaded since $m_d > \bar{k}$. Now suppose that the planner's choice is a vector (k, m_i), where $k \leq \bar{k}$ is the rate of deceased-donor transplantations, and m_i is the arrival rate of international patients. Since each international patient is accompanied by a non-directed donor, m_i is also the arrival rate of international non-directed donors. Let $\overline{m_i}$ be an exogenous upper bound on m_i, i.e. the maximum possible arrival rate for international patients.

Our analysis in D and E shows that an ϵ increase in m_i decreases the average waiting time of patients in the national queue by ϵ. This holds under a mild assumption: as long as the average length of the national queue does not become too small. Intuitively, when the length of the queue is moderately large, there are patients who exit the queue without getting transplanted; these patients have high dialysis costs. Increasing m_i by ϵ would increase the transplant rates for these patients, and this reduces their dialysis costs proportionally. This argument continues to hold until the queue becomes very short, i.e. when almost all patients are transplanted almost immediately. We refer the reader to D and E for details.

Remark 3.1. Although the qualitative results in both stochastic and fixed departure models are very similar, the analysis of these models is different since the fixed departure model is not Markovian. In the stochastic departure model, the expected changes in the pool size (at any moment) is only a function of the size of the pool; this enables us to write the mean-field equations. In the fixed departure model, however, we will not be able to write mean-field equations just in terms of the pool size, and so we have to take a different approach. To handle fixed departure times, we first analyze a simpler model: a model without abandonment in which each person never leaves the queue. This helps to establish a lower and upper bound on the average waiting time (per patient) in the original model. The upper and lower bound are close, and this provides a sharp estimate for the average waiting time in the original model.

Once we establish that an ϵ increase in m_i decreases the average waiting time of patients in the national queue by ϵ, it is straightforward to find the range for m_i for which the NDD proposal is self-financing: if (m_i, k) is a self-financing choice, then any other choice $(m_i+\epsilon, k)$ is also self-financing (as long as it satisfies our mild assumption). This characterizes each planner's choice as follows:

The Private Insurer's Choice. The private insurer's goal is minimizing the total cost. If the average dialysis cost of domestic patients who do not get transplants[14] is smaller than twice the surgery cost, then the planner does not use the NDD proposal, i.e. it sets $k = \bar{k}$ and $m_i = 0$.[15] (Since the NDD proposal increases total costs in this case) Otherwise, the

[14]So long as the average length of national queue is not "too small", this quantity is also (up to lower order terms) equal to the average dialysis cost of domestic patients when there are no transplant opportunities available, i.e. when $\bar{k} = 0$.

[15]The average dialysis cost of domestic patients who do not get transplants is currently well above twice the surgery cost: The average time that a patient can stay on dialysis without getting transplantation is about 5 years, the average dialysis cost per year per patient is about \$90,000, and the surgery cost per patient is about \$90,000. (Recall our estimates from Section 1.1)

planner chooses $k = \bar{k}$ and $m_i = \min\{\overline{m_i}, m_d - k\}$. The choice of k uses all the available deceased donors since they are less costly than international non-directed donors, and the choice of m_i either will employ all the available international non-directed donors, or will exhaust the waiting list.

The State Department. The State department's goal is to maximize the number of international patients subject to the constraint that at least as many domestic patients should be transplanted as in the status quo, and subject to the self-financing constraint. If the average dialysis cost of domestic patients who do not get transplants is smaller than twice the surgery cost, then the planner sets $k = \bar{k}$ and $m_i = 0$. (Since the NDD proposal increases total costs in this case) Otherwise, the system remains self-financing for any $m_i \leq m_d$, and therefore, the State department will employ as many international non-directed donors as possible, i.e. it chooses $k = \max\{0, m_d - \overline{m_i}\}$ and $m_i = \min\{\overline{m_i}, m_d\}$.

When the State department has a more relaxed financial constraint, namely, that the additional surgery costs for international patients should be covered by the reduction in the dialysis costs of domestic patients, then it will choose to use the NDD proposal whenever the average dialysis cost per domestic patient in the absence of transplants is smaller than the cost of a single surgery.

4 Repugnance Considerations

Transplantation of an organ from a living donor involves surgery on both a sick patient and a healthy one. The involvement of the healthy donor imposes special burdens on all involved, since one of the frequently invoked principles in medicine is "first, do no harm" (see [Rapaport and Starzl 1994, Smith 2005]).

Ever since the first successful solid organ transplant in 1954, from a living kidney donor (who lived until 2010), a great deal of thought and some controversy has concerned how to translate this principle to the care of donors, and to weigh the moderate risks that a healthy kidney donor assumes against the life-saving benefits that the donor wishes to achieve. The discussion of these issues at one time focused on whether living donation should be permitted at all, and if so from whom to whom. In the United States these questions have led to procedures for screening potential donors by evaluating their physical and mental health, and state of mind (e.g. whether they are in fact eager to donate) [Reese et al. 2015]. Some countries additionally restrict donation to be only between immediate family members.

(Germany is an example, and in consequence kidney exchange is generally not available in Germany.[16]) In this spirit, more stringent requirements have sometimes been established for screening non-directed donors, since the benefit to them from their donation is less direct than to a donor who saves the life of a family member.[17] And all of these questions arise with greater force when the donor appears vulnerable or exploitable.

To understand the kinds of concerns that will be raised about kidney exchange between developed and developing countries, it is worth noting the ongoing debate about whether it might be ethical to create legal markets in which kidneys for transplant could be purchased. With the single exception of the Islamic Republic of Iran, no country explicitly allows organs for transplant to be bought from or sold by living kidney donors for cash compensation. In most of the world such transactions are illegal, although there are black markets. These laws outlawing the purchase of kidneys are motivated largely by concern that legalizing such purchases might work to the disadvantage of poor and vulnerable people. Similar concerns will naturally arise when we think of kidney exchange involving donors from poor countries, whether those donors are in patient-donor pairs or are nondirected donors.

In fact, similar concerns arose when kidney exchange was initially proposed (but see [Ross et al. 1997]), and again when it became an operational reality. However, unlike a monetary market, kidney exchange did not arouse the kind of repugnance that led to legislation to prevent it. Quite the contrary: when the wording of the existing American law made it seem that it might preclude kidney exchange (as Germany's pre-existing laws do), Congress acted to clarify that kidney exchange was intended to be legal.

In the United States, the National Organ Transplant Act (NOTA) of 1984 specifies that "It shall be unlawful for any person to knowingly acquire, receive, or otherwise transfer any human organ for valuable consideration for use in human transplantation... ." This raised a potential barrier to kidney exchange, if a kidney in return for a kidney is viewed as "valuable consideration" of the kind precluded by the NOTA, although the transplant community received legal advice that encouraged them to go forward. As kidney exchange began to

[16] So GKE might be extended to German patient-donor pairs as well.

[17] Alongside the concern for protecting donors (and therefore allowing only the very healthiest donor candidates go forward), there has grown up a parallel set of concerns about protecting the rights of donors to save the lives of their loved ones, so that e.g. a patient with marginally high blood pressure who once would have been sent home might today successfully insist on being allowed to donate a kidney to his spouse. Similarly, many non-directed donors have made clear that being a donor is something that they feel called upon to do and that they benefit from. A noticeable fraction of non-directed donors in the U.S. come from faith based organizations, some of which are specifically organized to promote kidney donation, like Renewal (http://www.life-renewal.org/), or which take kidney donation to be an integral part of their religious calling, such as the Jesus Christians (http://www.jesuschristians.com/media-section/kidneys).

be performed on a wider scale, Congress amended the NOTA via the Norwood Act (Public Law 110-144, 2007), which said that the sentence about valuable consideration "does not apply" to kidney exchange.[18] The Norwood Act passed without any dissenting votes in either the House or the Senate (https://www.govtrack.us/congress/bills/110/hr710)[19]. This suggests that the repugnance attached to the purchase and sale of organs for transplant—reflected in the NOTA—did not carry over at all to kidney exchange.

In contrast, while GKE has received very significant initial support, it has also been attacked as repugnant. When [Rees et al. 2017b] describing the first Global Kidney Exchange chain was published in the *American Journal of Transplantation*, it was accompanied by an editorial published in the same issue suggesting that GKE might be repugnant ([Wiseman and Gill 2017]). Subsequently [Delmonico and Ascher 2017] claimed that ethical GKE with patient-donor pairs from the developing world "is not feasible when the culture is so experienced with organ sales." They argued that efforts to ban kidney sales have failed to such an extent that transplanting patients from poor countries would result in paid donors infiltrating American health care in contrived guises as spouses and relatives. Following various replies (see [Rees et al. 2017b] and [Roth et al. 2017]), the American Society of Transplant Surgeons issued an endorsement of Global Kidney Exchange,[20] and Italy endorsed it at the World Health Organization (see [ASTS 2017], [Emond 2017], and the Statement from Italy [21]). But this has not persuaded everyone, so more work needs to be done[22].

Global kidney exchange has also received very favorable reviews in the developing world countries in which it has been initiated. The second Global Kidney Exchange, conducted by the APD, involved a patient-donor pair from Mexico and was reported in Newsweek en

[18] Norwood Amendment of NOTA defines "human organ paired donation" as the donation and receipt of a human organ under the following circumstances: (1) an individual (the first donor) desires to make a living donation of a human organ specifically to a particular patient (the first patient), but such donor is biologically incompatible with such patient; (2) a second individual (the second donor) desires to make a such a donation to a second patient, but is biologically incompatible with such patient; (3) the first donor is biologically compatible to the second patient, and the second donor is biologically compatible to the first patient; (4) all donors and patients enter into a single agreement to donate and receive such organs; and (5) no valuable consideration is knowingly acquired, received, or otherwise transferred with respect to the organs.

[19] The vote in the House, on March 7, 2007 had 422 "yea" votes, 0 "nay" votes and 11 members were recorded as "not voting"). The subsequent vote in the House to reconcile the bill with the Senate version was taken (on December 4) under a procedure called "suspension of the rules" which is typically used to pass non-controversial bills. The vote on the "Motion to Suspend the Rules and Agree in the House" received 407 "yea" votes and 1 "nay".

[20] http://marketdesigner.blogspot.com/2017/10/global-kidney-exchange-endorsed-by.html

[21] http://apps.who.int/gb/statements/eb142/PDF/22/Italy-3.1.pdf

[22] Many of the specific objections that have been raised to GKE are discussed in [Bozek et al. 2018]

Espanol ([Carrillo 2017]) with a cover story titled “Un Puente de Vida,” “A Bridge of Life.” It opened by applauding GKE by saying (via Google Translate) ”At the same time that US President Donald Trump is seeking to build a wall of thousands of miles on the border with Mexico, a tireless surgeon and a renowned economist join forces to exchange organs between citizens of both countries.”[23]

There is thus reason to be cautiously optimistic that global kidney exchange may also overcome potential objections, in both the rich and poor nations that would be involved.

Notice that if an international exchange works perfectly—i.e. when all of the patients and donors involved have successful surgeries, excellent follow-up care, and are all restored to active, long-lasting good health—then it will be easy to see the exchange as just another example of the success of standard kidney exchange in which all patients are from the same country. But if the pair from the poor country were to return home and have bad health outcomes, it would look a lot like the badly arranged black market transactions, which are justly condemned. So to make kidney exchange work between developed and developing countries, exceptional care will have to be delivered to the developing-country donors and patients, particularly since patients in poor countries–like their compatriots who have never suffered from kidney disease–can be expected to have somewhat worse health outcomes than otherwise comparable people in rich countries, no matter what efforts are made to give them the best possible post-operative care.[24]

These ethical/moral repugnance concerns lead to a number of practical design issues, which may play out differently in different countries.

First, the developing country will need a substantial medical infrastructure before it can reliably provide care for returning transplant patients and donors. This is one reason why initial pilot exchanges have begun between the U.S. and the Philippines (and the U.S. and Mexico), where the infrastructure for transplantation and postoperative care already exists, although treatment is beyond the means of a significant part of the population. In contrast, developing kidney exchange with Nigeria will depend on first establishing infrastructure not only to care for post-surgical patients, but to identify and care for patients with both

[23]Similarly, when one of the authors (AER) presented an outline of this proposal at Covenant University in Nigeria, no issues of repugnance were raised, and two of the five Nigerian discussants said something to the effect of “Now I understand why G-d has given us two kidneys.”

[24]As of this writing (in June 2018) all of the foreign GKE patients and donors are in good health. The initial patient-donor pair from the Philippines, transplanted in January 2015, is presently in good health and receiving post-operative care as needed at home in the Philippines, funded from a $50,000 escrow fund. The Mexican patient-donor pairs are under the care of Mexican physicians, whose costs are covered by Mexican health insurance.

chronic and end stage renal disease, so that candidates for exchange can even be identified in a systematic way. The requirements for developing infrastructure and financing continued care are closely related to the design of the financial flows, to be discussed below.

A second repugnance issue is that, at least so long as it remains a crime in the participating countries to purchase a kidney for transplant, the ethical and legal solicitation of foreign donors will have to be addressed. (In the United States, this concern sometimes arises for unrelated directed donors who have not had extensive prior connections to the intended recipient). For directed donors, some countries may wish, at least initially, to adopt practices like those in Germany, which restrict donation to family members, or perhaps to others with whom the donor has a demonstrable connection (e.g. members of the same religious congregation.)

For non-directed donors, designing exchange to prevent illegal compensation of donors may be more complicated. In the United States, non-directed donors who present themselves as such make their donation to an anonymous recipient.[25] This practice could be adopted in the case of foreign non-directed donors in one of two ways. It could be required that the foreign non-directed donor should not be told the identity of either the American recipient nor the foreign patient-donor pair whose transplant would be facilitated. In a large enough pool, this would effectively prevent the purchase of the non-directed donor's kidney by the pair that would then be invited to receive a transplant financed by American healthcare savings. But it might also deter some donors who wished to make a donation to facilitate the transplant of a particular pair. So it might also be desirable to enlarge the definition of what it means to be a non-directed (bridge) donor in global kidney exchange to include a donor who wishes his donation to an American to benefit a specific patient-donor pair in his own country. Once again, some countries might want to require that both donors (the donor who will donate to the developing country patient, and the donor who will donate to an American on the waiting list) be demonstrably related to the developing-country kidney patient, at least until concerns about possible payments could be assuaged in some other way.

Note that, if our concern in this paper were only with American patients, global kidney exchange, with its costs of care for international patients, would likely be more expensive than other ways of increasing the number of donor kidneys available to Americans. (These avenues should also be pursued, of course). Some of these—like increasing the number of

[25] As noted earlier, some non-directed donors may present themselves to kidney exchange as unrelated directed donors, after having been matched with a candidate by the transplant organization to which they first presented themselves.

deceased donor kidneys—would not cover the full need for organs, but each viable organ is very valuable. Other avenues, like increasing the number of American donors by providing greater incentives to donate, may be repugnant and illegal under current American law. It seems likely that financial disincentives to donation could be reduced under current law, however. Each of these avenues is well worth exploring, and each has the prospect of helping save American lives and medical costs, but none of them would offer the prospect of extending the benefits of transplantation to international patients while accomplishing the domestic American goals.

This brings us back to the practicalities of financing kidney exchanges which include foreign patients and donors. It is not enough to observe that the process saves money because transplantation is so much cheaper than dialysis. It is also necessary to think about how these savings can be turned into payments to cover the different pattern of costs that would be incurred. In the United States, where each medical procedure often has to be individually paid for, new financial flows will be needed to cover the costs that transplant centers incur in transplanting international patients.[26] New financial flows will be needed in the developing world, to cover both the direct costs of post-surgical care, and the costs of developing the infrastructure to provide care to the developing-world recipients and donors, not only after surgery in the United States, but also beforehand, during the process of identifying candidates and matching them as necessary to American patient-donor pairs or waiting list recipients. The proposals considered here can also be used to increase the surgical infrastructure in the developing world, as the logistics are organized to allow some of the GKE and NDD surgeries involving developing world patients and donors to be financed in the U.S. but conducted in the developing world.[27]

5 Designing the financing

The question of how to deploy the savings from transplantation in developed countries to fund the costs of transplantation for citizens of developing nations will be central to realizing the potential of global kidney exchange to be self-funding. The design of these financial flows may be among the most difficult design issues facing these proposals. (Even as kidney

[26]The design of financial flows may be easier in host countries with single-payer healthcare systems, in which the savings from dialysis are accrued by the same organization that pays for the relevant surgeries.

[27]In Mexico, the initial global kidney exchanges involving Mexican patients has led to the establishment of a domestic kidney exchange clearinghouse called Pro-Renal [see http://www.pro-renal.com/alianzas/global-kidney-exchange/ and https://marketdesigner.blogspot.com/2018/05/kidney-exchange-takes-another-step.html].

exchange has grown in the United States, some aspects of the financial transactions between hospitals and payers continue to cause unnecessary frictions, see e.g.[Rees et al. 2012], but because there are already financial mechanisms in place to fund surgeries, these difficulties have mostly been surmountable.)

Since much of the savings from global kidney exchange in the U.S. would accrue to Medicare, a natural thought is that payments to cover the foreign patients and donors would come from Medicare. But Medicare is a complicated mix of legislation and bureaucracy, and the project of allowing new costs to be billed to Medicare—costs involving foreign patients—might be too difficult to achieve.[28]

The new costs of surgery and postoperative care for patients and donors will fall on individual transplant centers in both the developed and developing world. While Medicare has well established ways of reimbursing transplant centers for the care of patients with Medicare coverage, none of the foreign patients and donors will fall under Medicare's responsibility. So without a substantial change in Medicare's payment authority—which is not impossible, but would involve substantial new legislation—we do not anticipate that the savings to Medicare can easily be translated into payments from Medicare to the transplant centers for the care of foreign patients and donors.

Other avenues for payments may be easier to arrange. Recall that, for patients who have private insurance in the U.S., the first 33 months of dialysis are covered by their insurer. Consequently, the savings to private insurers and particularly to self-insuring companies from transplanting a patient about to begin dialysis could fund the expenses of the foreign patients (e.g. the costs of all evaluation, preparation and surgery, and a fund held in escrow to pay for future care at home[29]).This particular flow of funds might involve prioritising patients to be included in an exchange based in part on insurance status and expected savings. This might also arouse some repugnance, since these financial considerations presently play no role in kidney exchange matching algorithms, yet are the source of the savings that may be easiest to access to make it possible to include foreign patient-donor pairs in American exchange.

[28]Even for American patients, Medicare payments are often restricted in counterproductive ways. For example, for covered patients Medicare pays the full cost of dialysis, and transplantation, but only pays for three years of immunosuppressive drugs following transplantation. A small number of patients, after running out of Medicare drug coverage, therefore lose their transplanted kidney and go back on dialysis, which Medicare then pays for, at much greater cost than that of the immunosuppressive drugs. (See e.g. http://marketdesigner.blogspot.com/2013/03/federal-budgets-and-immunosuppressive.html)

[29]For American patients, insurance for adverse outcomes related to either the recipient's transplantation or their living donor's nephrectomy is included in the transplant recipient's medical insurance or Medicare coverage. Adverse outcome insurance for foreign patients and donors will be an additional cost.

Infrastructure for caring for overseas patients in their home countries could perhaps be funded differently. Presently, the U.S. government, through USAID, helps developing nations combat infectious diseases (as well as some non-infectious disease health issues such as maternal and child health, nutrition—see Bureau for Global Health of the United States Agency for International Development, 2015). Support for global kidney exchange would fit well with the kind of enlightened self-interest that often helps build domestic coalitions in support of foreign aid: like aid to fight infectious diseases, it would address a pressing health issue in the recipient countries while at the same time providing some benefits to the donor country. Notice that the self-financing aspect of global kidney exchange would continue to apply to funds disbursed through USAID, even if Medicare itself cannot disperse those funds, since the savings to Medicare are savings to the same overall Federal budget that finances USAID. So funding through another part of the Federal budget would remove the need to dis-assemble and reassemble parts of Medicare protocol and authorizing legislation that might be too politically complicated to accomplish.

6 Concluding Remarks

Kidney transplants, which are presently the best treatment for end-stage kidney disease, are also the cheapest, and so including the developing world in the kidney exchanges that are becoming standard in many parts of the developed world offers the possibility of mutual aid. This is because transplantation is limited in both the developing and developed world, but for different reasons. In the developing world, resources for this kind of advanced medical treatment are in short supply. In the developed world, organs are in short supply. Kidney exchange already offers patients and those who love them the opportunity to receive transplants in ways that increases the supply of organs, and we have argued that extending these benefits to countries without the financial resources for transplantation can be self-financing.

We consider two proposals. The first, Global Kidney Exchange, which involves including only patient-donor pairs from the developing world, has already begun to be implemented on a very small scale ([Rees et al. 2017a] and [Carrillo 2017], which has shown that the logistics are practical. We show here that this proposal could be self financing even when conducted on a large enough scale to reduce waiting times and hence dialysis costs for American patients. The second, new proposal additionally involves non-directed donors from the developing world who wish to facilitate the transplant of a fellow citizen. In both cases we show that these procedures would be self-financing in the steady state up to the point when the average cost of dialysis is not smaller than the cost of surgery, and for the GKE proposal, even when carried to the point that the waiting time for American patients is reduced so much that the average cost of dialysis becomes less than the cost of surgery, because the American patients who would be matched to foreign pairs would tend to be those with higher than average dialysis costs.

We identify two main obstacles to practical implementation on a large scale. The first involves engineering the financial flows so that the new costs of care for foreign patients and donors could in fact be financed out of the (larger) savings from taking American patients off dialysis. The second involves addressing–in ways that command wide support and that avoid or ameliorate possible sources of repugnance–the ethical concerns that will arise in dealing with patients and donors from poorer countries, and making sure that they receive a level of care and of success comparable to the American patients and donors with whom they will exchange. We anticipate that it is feasible to satisfactorily resolve both of these issues.

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Appendicies

A Comparison to a Simple Static Model

We set up and analyze a very simple static model and compare it with the dynamic steady-state model that we discussed earlier. There is a mass m of domestic pairs. The mass of domestic pairs who find and exchange with a match within this pool is determined by matching function $\rho : \mathbb{R}_+ \to [0, 1]$; suppose $\rho(m)$ is the mass of such pairs. The transplant cost per unit mass of pairs is s. The unmatched pairs (with mass $m - \rho(m)$) would go under dialysis, the cost of which is d per unit mass of pairs.

The planner is given an option of choosing a parameter $\lambda \in [0, \overline{\lambda}]$ and letting a fraction λm of international pairs to participate in the market in the following way. After the domestic exchanges are made, the planner searches for exchange possibilities between international pairs and domestic pairs. Suppose that a matching function $\eta(x, y)$ determines the fraction of domestic pairs who exchange with a compatible international pair at the end of the search process, where x, y denote the mass of participating pairs in the domestic and international pools, respectively.

Total health care costs as function of λ can then be written as

$$\mathbf{C}(\lambda) = s \cdot \Big(\rho(m) + \eta(m - \rho(m), \lambda m)\Big) + d \cdot \Big(m - \rho(m) - \eta(m - \rho(m), \lambda m)\Big).$$

Suppose the planner's problem is finding $\lambda \in [0, \lambda^*]$ to minimize $\mathbf{C}(\lambda)$. We denote the minimizer by λ^*. It is not hard to see that if $d < 2s$, then $\lambda^* = 0$, and otherwise, $\lambda^* = \overline{\lambda}$.[30] In other words, let $\overline{C}_{\text{sta}}$ denote the expected healthcare costs for a domestic pair conditioned on not finding a domestic match. Then, $\lambda^* > 0$ iff $\overline{C}_{\text{sta}} \geq 2s$.

Does the same insight carries over in the dynamic model? The answer is negative. In what follows, we go over two aspects of the cost-benefit analysis that are missed in the static model, but captured in the dynamic model. To understand these aspects better, we go through a thought experiment where the planner is investigating whether increasing the (previously set) value of λ would decrease the total healthcare costs.

The first missing aspect in the static analysis is that it does not consider the "option-value" of the domestic pairs. By allowing an exchange between a domestic and an international pair, the planner could be eliminating a future exchange between domestic pairs. In other words, by increasing λ the planner is making the domestic pool thinner, which could

[30] This holds under natural monotonicity conditions on ρ, η: they should be increasing in their arguments.

reduce the number of future domestic exchanges. This explains why the answer to the question in the last paragraph is negative. To be more precise, let $\overline{C}_{\text{dyn}}(\lambda)$ denote the expected healthcare costs of a domestic pair (in the dynamic model) conditioned on not finding a match upon arrival. Even when the condition $\overline{C}_{\text{dyn}}(\lambda) > 2s$ holds in the dynamic model, increasing λ could increase the total healthcare costs, $\mathbf{C}(\lambda)$. As we explained earlier, the reason is the option-value of keeping the domestic pairs in the pool.

The second aspect is that the cost-benefit analysis of the static model only accounts for the expected dialysis cost of unmatched pairs, unlike the analysis of the dynamic model which also accounts for the role of expected surgery costs. As we elaborate next, this comes with significant consequences. Let us write

$$\overline{C}_{\text{dyn}}(\lambda) = \overline{D}_{\text{dyn}}(\lambda) + \overline{S}_{\text{dyn}}(\lambda) + \overline{S}_{\text{dyn,i}}(\lambda),$$

where $\overline{D}_{\text{dyn}}(\lambda), \overline{S}_{\text{dyn}}(\lambda)$ and $\overline{S}_{\text{dyn,i}}(\lambda)$ respectively denote for a domestic pair who does not find a match upon arrival, its expected dialysis cost, its expected surgery cost, and the expected surgery cost paid for an international pair who participates in an exchange with this domestic pair. The term $\overline{D}_{\text{dyn}}(\lambda)$ is decreasing in λ, intuitively because patients in the domestic pool spend less time on dialysis for higher λ. This creates a force that works against increasing λ. On the other hand, the term $\overline{S}_{\text{dyn,i}}(\lambda)$ is increasing in λ, intuitively because the chance of an exchange with an international pair goes up with λ.[31] As this chance goes up, the planner becomes more willing to arrange an earlier exchange with an international pair to avoid the dialysis cost of the domestic pair. This creates a force that works in favor of increasing λ (which we discussed also earlier). Such a force is absent in the static model. It is noteworthy that since the discussed forces push λ in opposite directions, the optimal values of λ in the dynamic and static model would be incomparable in general, i.e., neither is an upper or lower bound for the other one.

B Uniqueness and differentiability of solutions

We prove the following lemmas before proving Proposition 2.1.

Lemma B.1. *The solution to* (2.1) *and* (2.2) *is unique when* $m > 0$ *and* $0 < \lambda < 1$.

Proof of Lemma B.1. We use a change of variables to simplify notation. Let $X = e^{-\gamma x}$ and

[31] It is true that $\overline{S}_{\text{dyn}}(\lambda) + \overline{S}_{\text{dyn,i}}(\lambda)$ also is increasing in λ. We based our argument on the second summand only for expositional simplicity.

$Y = e^{-\gamma y}$. This lets us rewrite (2.1) and (2.2) respectively as

$$XY = (1+\lambda)(1-X) - \frac{\log X}{m\gamma}, \tag{B.1}$$

$$\lambda X = (1-Y)X - \frac{\log Y}{m\gamma}. \tag{B.2}$$

Let (X_m^*, Y_m^*) denote the solution to the system of equations given by (B.1) and (B.2) (for a particular value of m). Later we will prove the existence and uniqueness of such solution.

We start by finding X in terms of Y. From (B.2) we get

$$X = -\frac{\log Y}{m\gamma(Y+\lambda-1)}. \tag{B.3}$$

Plugging the above equation in (B.1) and rearranging gives

$$\begin{aligned}&(-\log Y)\cdot(1+\lambda+Y)\\ &= m\gamma\cdot(1+\lambda)(Y+\lambda-1) - \log\left(-\frac{\log Y}{m\gamma(Y+\lambda-1)}\right)(Y+\lambda-1).\end{aligned} \tag{B.4}$$

Claim B.2. *Equation (B.4) has a unique solution, which we denote by Y_m^*.*

Proof. Note that (B.3) is suggesting that $Y > 1-\lambda$, since $X < 0$ otherwise. (Also, (B.4) is well-defined only when $Y > 1-\lambda$.) We conjecture that this is always the case and proceed the proof assuming that $Y > 1-\lambda$. In the end, the proof confirms that this is indeed the case.

Let $L(Y), R(Y)$ be functions denoting the left-hand side and the right-hand side of (B.4) (as a function of Y), respectively. We will show that $L(Y)$ is strictly decreasing in Y and $R(Y)$ is strictly increasing in Y. Then, we will use this to prove the existence and the uniqueness of the solution, Y_m^*.

$L(\cdot)$ is strictly decreasing. We show that $L(Y)$ is strictly decreasing in Y, for $Y \in (1-\lambda, 1)$. To this end, we will show that $L'(Y) > 0$. See that

$$-L'(Y) = 1 + \frac{1+\lambda}{Y} + \log Y.$$

It is straight-forward to verify that the right-hand side of the above equation is a concave function of Y, and therefore, the claim is proved if we show that both $-L'(1-\lambda) > 0$ and

$-L'(1) > 0$ hold. Note that $-L'(1) = 2 + \lambda > 0$. It remains to show that $-L'(1-\lambda) > 0$. See that

$$-L'(1-\lambda) = 1 + \frac{1+\lambda}{1-\lambda} + \log(1-\lambda).$$

Now, note that

$$-\log(1-\lambda) = \log\frac{1}{1-\lambda} < \frac{1}{1-\lambda},$$

since $z < e^z$ for all z. Therefore, $\frac{1+\lambda}{1-\lambda} + \log(1-\lambda) > 0$, which means that $-L'(1-\lambda) > 0$. The concavity of $-L'$ then implies that L is strictly decreasing in Y, for $Y \in (1-\lambda, 1)$.

$R(\cdot)$ is strictly increasing. To prove this, we write

$$\begin{aligned} R(Y) &= (Y+\lambda-1)\cdot\left(m\gamma\cdot(1+\lambda) - \log\left(-\frac{\log Y}{m\gamma(Y+\lambda-1)}\right)\right) \\ &= (Y+\lambda-1)\cdot\left(m\gamma\cdot(1+\lambda) - \log(-\log Y) + \log(m\gamma) + \log(Y+\lambda-1)\right). \end{aligned} \tag{B.5}$$

From this representation it becomes clear that, since the terms $Y+\lambda-1$ and $-\log(-\log Y)$ are increasing in Y, then the function $R(Y)$ is also increasing in Y.

Next, we use the fact that $L(\cdot)$ and $R(\cdot)$ are strictly monotone (i.e. strictly decreasing and strictly increasing, respectively) to prove that (B.4) has a unique solution. Note that if (B.4) has a solution, its uniqueness is guaranteed by strict monotonicity. It remains to show the existence of the solution.

The functions $L(\cdot)$ and $R(\cdot)$ cross. Next, we show that there exists $Y_m^* \in (1-\lambda, 1)$ which solves (B.4). A straight-forward calculation shows that

$$\begin{aligned} \lim_{Y\to(1-\lambda)^+} L(Y) &= -2\log(1-\lambda) > 0, \\ \lim_{Y\to(1)^-} L(Y) &= 0, \\ \lim_{Y\to(1-\lambda)^+} R(Y) &= 0, \\ \lim_{Y\to(1)^-} R(Y) &= +\infty. \end{aligned}$$

The above limits, together with the strict monotonicity of the functions L, R prove the claim. $\square$

Given the uniqueness of Y_m^*, the uniqueness of X_m^* is implied by (B.3). $\square$

Lemma B.3. *The solution to (2.1) and (2.2) is unique when $m > 0$ and $\lambda = 0$.*

Proof. Observe that, at $\lambda = 0$, the LHS of (2.2) is equal to 0. (2.2) then implies that y must be equal to 0. The system of equations then reduces to

$$m\overline{\mu}(x) = m\mu(x) + x.$$

Denote the left-hand and right-hand side of the above equation (as a function of x) by $L(x), R(x)$, respectively. Observe that $L(x)$ is strictly decreasing, while the $R(x)$ is strictly increasing. Therefore, uniqueness of the solution is guaranteed when a solution exists. To prove existence, observe that

$$\begin{aligned} L(0) &= m, \\ \lim_{X\to\infty} L(X) &= 0, \\ R(0) &= 0, \\ \lim_{X\to\infty} R(X) &= +\infty. \end{aligned}$$

The above equations, together with continuity of L, R prove the existence of at least one solution to the system of equations. This concludes the proof. $\square$

Proof of Proposition 2.1. The uniqueness of solution is implied by Lemma B.1 and Lemma B.3. It remains to prove that the functions $x(\lambda), y(\lambda)$ are continuously differentiable (in λ). The proof is done by applying the Implicit Function Theorem. First, we rewrite the system defined by (2.1) and (2.2) as

$$F(\lambda, x, y) = (0, 0), \tag{B.6}$$

where $F : \mathbb{R}^3 \to \mathbb{R}^2$ is defined as

$$F(\lambda, x, y) \equiv \big(F_1(\lambda, x, y), F_2(\lambda, x, y)\big),$$

with

$$\begin{aligned} F_1(\lambda, x, y) &\equiv m\overline{\mu}(x)\overline{\mu}(y) - m(1+\lambda)\mu(x) - x, \\ F_2(\lambda, x, y) &\equiv \lambda m\overline{\mu}(x) - m\overline{\mu}(x)\mu(y) - y \end{aligned}$$

Note that by Lemma B.3, there exists a unique solution satisfying (B.6). Second, observe that the Jacobian

$$J_{F,(x,y)}(\lambda, \mathbf{x}(\lambda), \mathbf{y}(\lambda)) = \begin{pmatrix} \frac{\partial F_1}{\partial x}(\lambda, \mathbf{x}(\lambda), \mathbf{y}(\lambda)) & \frac{\partial F_1}{\partial y}(\lambda, \mathbf{x}(\lambda), \mathbf{y}(\lambda)) \\ \frac{\partial F_2}{\partial x}(\lambda, \mathbf{x}(\lambda), \mathbf{y}(\lambda)) & \frac{\partial F_2}{\partial y}(\lambda, \mathbf{x}(\lambda), \mathbf{y}(\lambda)) \end{pmatrix}$$

is invertible at any $\lambda \in [0, 1]$. We prove this by showing that the determinant of the matrix

$$J_{F,(x,y)}(\lambda, \mathbf{x}, \mathbf{y}) = \begin{pmatrix} \frac{\partial F_1}{\partial x}(\lambda, \mathbf{x}, \mathbf{y}) & \frac{\partial F_1}{\partial y}(\lambda, \mathbf{x}, \mathbf{y}) \\ \frac{\partial F_2}{\partial x}(\lambda, \mathbf{x}, \mathbf{y}) & \frac{\partial F_2}{\partial y}(\lambda, \mathbf{x}, \mathbf{y}) \end{pmatrix}$$

is equal to

$$e^{-2\gamma(\mathbf{x}+\mathbf{y})} \left(2\gamma^2 m^2 e^{\gamma \mathbf{y}} + \gamma(\lambda+1) m e^{\gamma(\mathbf{x}+\mathbf{2y})} + 2\gamma m e^{\gamma(\mathbf{x}+\mathbf{y})} + e^{2\gamma(\mathbf{x}+\mathbf{y})}\right),$$

which is always greater than 0 for any $\lambda \in [0, 1]$. For brevity, we exclude the algebraic computations for computing this determinant; these calculations are done in File "GKE-det.m".

For any $\lambda \in [0, 1]$, the implicit function theorem therefore implies that there exists an open set $I = (\alpha, \beta)$ containing λ and a unique continuously differentiable function $g : \mathbb{R} \to \mathbb{R}^2$ such that $F(\lambda', g(\lambda')) = 0$ for all $\lambda' \in I$.[32] This fact, together with Lemma B.1 and Lemma B.3, conclude the proof. □

C Proofs for Theorems 2.2 and 2.3

Proof of Theorem 2.2. Let d be the average dialysis cost per patient per unit of time and s denote the surgery cost per patient. The proof is based on the balance equations. Recall that:

$$m\overline{\mu}(x)\overline{\mu}(y) = m(1+\lambda)\mu(x) + x, \tag{C.1}$$

$$\lambda m\overline{\mu}(x) = m\overline{\mu}(x)\mu(y) + y. \tag{C.2}$$

[32] When applying the Implicit Function Theorem on the endpoints of the interval $[0, 1]$, we should extend the domain of F in the natural way to allow for values of λ slightly lower and higher than 1, respectively.

Let $x(\lambda), y(\lambda)$ denote the solution to the above system as a function of λ. Therefore, the total healthcare costs could be written as

$$\mathbf{C}(\lambda) = d \cdot x(\lambda) + s \cdot \Big[(\lambda+1)m - (x(\lambda) + y(\lambda))\Big].$$

We then can write

$$\begin{aligned}\mathbf{D}(\lambda) &\equiv \mathbf{C}(\lambda) - \mathbf{C}(0) \\ &= d \cdot (x(\lambda) - x(0)) + s \cdot \Big[\lambda m - x(\lambda) - y(\lambda) + x(0)\Big]\end{aligned}$$

From the above equation, we can compute the derivative of $\mathbf{D}(\lambda)$ at $\lambda = 0$.

$$\mathbf{D}'(0) = d \cdot x'(0) + s \cdot [m - x'(0) - y'(0)].$$

We simplify the above equation further. Let θ be the ratio of average dialysis cost per person to the average surgery cost per person at $\lambda = 0$, i.e. $\theta = \frac{\frac{x(0)d}{m}}{s}$. We therefore can rewrite the above equation in terms of θ, s as follows

$$\mathbf{D}'(0) = \theta s m \cdot \frac{x'(0)}{x(0)} + s \cdot [m - x'(0) - y'(0)]. \tag{C.3}$$

In the rest of the proof, we will show that $\mathbf{D}'(0) < 0$, for all $m > m_\gamma$, where m_γ is a constant that will be set in the end of the proof. The proof is straight-forward. We compute closed-form expressions for $x'(0)$ and $y'(0)$. This is done by taking total derivatives from (C.1) and (C.2) with respect to λ, and solving the resulting 2×2 system of equations. While it is possible to do this manually, we use Mathematica to derive the closed-form expressions. Our calculations are summarized in the file "GKE-1.m". To avoid heavy notation, we plug the solutions for $x'(0)$ and $y'(0)$ directly into (C.3), which results in the following expression:

$$\begin{aligned}&\frac{x\mathbf{D}'(0)}{s} = \\ &x\left(-\frac{m}{\gamma m + e^{\gamma x}} + \frac{m e^{\gamma x}\left(\gamma m + e^{\gamma x} - 1\right)}{\left(\gamma m + e^{\gamma x}\right)\left(2\gamma m + e^{\gamma x}\right)} + m\right) - \frac{\theta m^2 e^{\gamma x}\left(\gamma m + e^{\gamma x} - 1\right)}{\left(\gamma m + e^{\gamma x}\right)\left(2\gamma m + e^{\gamma x}\right)}.\end{aligned} \tag{C.4}$$

where we have used x to denote $x(0)$ to simplify notation. To finish the proof, note that first term in the right hand side is $xm + o(xm)$, and the second term in the right-hand side is

$$-\Big(\frac{\theta m e^{\gamma x}}{2\gamma} + o\left(\frac{\theta m e^{\gamma x}}{2\gamma}\right)\Big).$$

(To make this observation, it is crucial to note that $e^{\gamma x} = o(m)$.) Therefore, the right-hand side of (C.4) is always negative for large enough m when $x < \frac{\theta e^{\gamma x}}{2\gamma}$, or equivalently, when

$$\frac{2\gamma x}{e^{\gamma x}} < \theta.$$

Now, note that

$$\lim_{m\to\infty} e^{-\gamma x} = \frac{1}{2}, \qquad \lim_{m\to\infty} x = \gamma^{-1}\cdot \ln(2),$$

which imply

$$\lim_{m\to\infty} \frac{2\gamma x}{e^{\gamma x}} = \ln(2). \tag{C.5}$$

Therefore, the right-hand side of (C.4) is negative for large enough m, whenever $\theta > \ln(2)$. In other words, we can set m_γ so that the right-hand side of (C.4) is negative for all $m > m_\gamma$, whenever $\theta > \ln(2)$.

The tightness of constant $\ln(2)$ is implied directly from the above analysis: the right-hand side of (C.4) is negative for sufficiently large m if θ is a constant smaller than the right-hand side of (C.5). □

Next, we state and prove a more general theorem than Theorem 2.3. We will verify that Theorem 2.3 is a direct corollary of the following theorem.

Theorem C.1. *Fix $\gamma > 0$. Then, for any $m > 0$ there exists a strictly decreasing and continuous function $f_m : \mathbb{R}_+ \to \mathbb{R}$ such that for any $\lambda \in [0,1]$, $\mathbf{C}'(\lambda) < 0$ holds iff $\Theta(\lambda) > f_m(\lambda)$. Furthermore, for any constant $\epsilon > 0$, there exists a constant m_ϵ such that for all $m > m_\epsilon$, $f_m(0) < \ln(2) + \epsilon$.*

We will provide an example and an intuitive discussion of Theorem C.1 after stating the proof. We will also discuss a natural economic interpretation for the function f_m: it is in fact equal to the marginal change (for a change in λ) in the average number of surgeries per domestic pair to the semi-elasticity (with respect to λ) of the average dialysis time per domestic pair.

Proof of Theorem C.1. The proof is quite similar to the proof of Theorem 2.2. We compute $\mathbf{C}'(\lambda)$ for all λ. ($\mathbf{C}'(0)$ is defined as the right derivative, which would coincide with $\mathbf{D}'(0)$, used in the proof of Theorem 2.2)

This is done by taking total derivatives from (C.1) and (C.2) with respect to λ and solving the resulting 2×2 system of equations. This gives

$$\mathbf{C}'(\lambda) = \frac{m\left(\gamma m + e^{\gamma(x_\lambda + y_\lambda)} - e^{\gamma y_\lambda}\right)\left(e^{\gamma x_\lambda}(2x - m\theta) + 2\gamma m x\right)}{x\left(2\gamma^2 m^2 + \gamma(\lambda+1)me^{\gamma(x_\lambda + y_\lambda)} + 2\gamma m e^{\gamma x_\lambda} + e^{\gamma(2x_\lambda + y_\lambda)}\right)}.$$

where we use x to denote $x(0)$ and use x_λ, y_λ to denote $x(\lambda), y(\lambda)$. For brevity, we suppress the algebraic calculations that derive this equality. The Mathematica file "GKE-2.m" contains the details. Observe that the denominator of the right-hand side is always positive. The numerator is negative iff

$$\left(e^{\gamma x_\lambda}(2x - m\theta) + 2\gamma m x\right) < 0.$$

Equivalently, the numerator is negative iff

$$\theta > \frac{2\gamma x}{e^{\gamma x_\lambda}} + \frac{2x}{m}.$$

Using the fact that $\Theta(\lambda) = \theta \cdot \frac{x_\lambda}{x}$, we can rewrite the above condition as

$$\Theta(\lambda) > \frac{2\gamma x_\lambda}{e^{\gamma x_\lambda}} + \frac{2x_\lambda}{m}.$$

The function $f(\lambda)$ (defined in the theorem statement) is then defined as the right-hand side of the above inequality, i.e.

$$f(\lambda) \equiv \frac{2\gamma x_\lambda}{e^{\gamma x_\lambda}} + \frac{2x_\lambda}{m}. \tag{C.6}$$

It remains to prove that the claimed properties for f hold. To see why $\lim_{m\to\infty} f(0) = \ln(2)$, recall (C.5) from the proof of Theorem 2.2, where we essentially prove this claim. It remains to show that

It remains to show that f is strictly decreasing in λ. There are two summands in the right-hand side of (C.6). The second summand is clearly strictly decreasing in λ. It remains to prove that the first summand, which we denote by $g_m(\lambda)$, is strictly decreasing in λ. We simply take the derivative with respect to λ. From "GKE-3.m", we have that

$$g'(\lambda) = \frac{m(\gamma x_\lambda - 1)\left(\gamma m + e^{\gamma(x_\lambda + y_\lambda)} - e^{\gamma y_\lambda}\right)}{2\gamma^2 m^2 + \gamma(\lambda+1)me^{\gamma(x_\lambda+y_\lambda)} + 2\gamma m e^{\gamma x_\lambda} + e^{\gamma(2x_\lambda + y_\lambda)}}$$

Observe that $g'(\lambda) < 0$ iff $\gamma x_\lambda - 1 < 0$. But this condition is always satisfied because

$$\gamma x_\lambda - 1 \leq \gamma x - 1 < \ln(2) - 1.$$

To see why the last inequality holds, recall the balance equation $m(2e^{-\gamma x} - 1) = x$, which implies $2e^{-\gamma x} - 1 > 0$, and therefore $\gamma x < \ln(2)$. $\square$

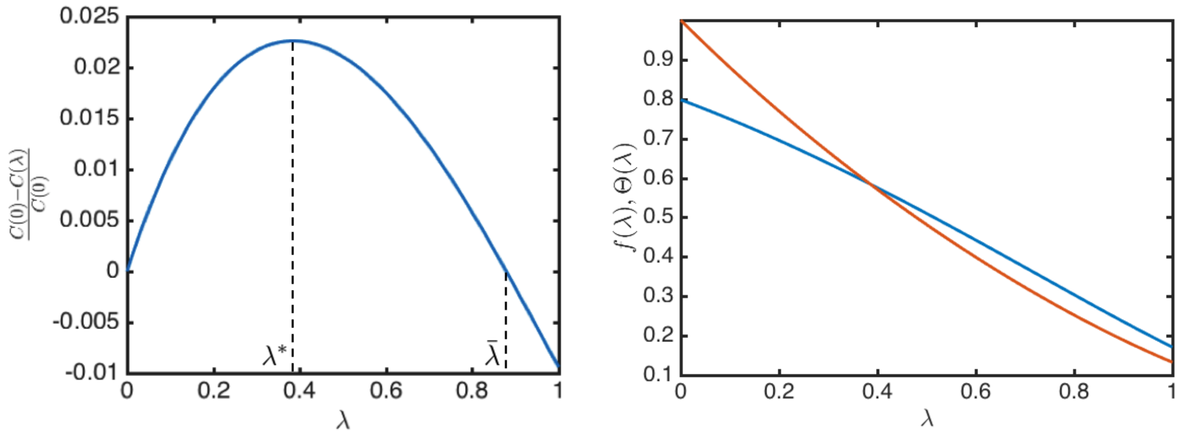
Proof of Theorem 2.3. The proof follows from Theorem C.1 and the fact that f_m is a decreasing function. $\square$

Next, we provide an example that helps to better understand Theorem C.1.

Example Figure 4 illustrates statistics concerning an instance of the problem with $m = 10, \gamma = 1, \theta = 1$. The percentage of reduction in total healthcare costs in terms of λ is given in Figure 4(a). Figure 4(b) demonstrates how the interaction of $\Theta(\lambda)$ with $f_m(\lambda)$ is related to $\mathbf{C}'(\lambda)$. At $\lambda = 0$, $\Theta(0) = 1$ and $f_m(0) = 0.8$, and therefore by Theorem C.1, $\mathbf{C}'(0) > 0$ must hold because $\Theta(0) > f_m(0)$. This is indeed the case, as shown in Figure 4(a). As λ increases, $\Theta(\lambda)$ decreases, but so does $f_m(\lambda)$. These functions intersect at $\lambda^* \approx 0.4$, where $\mathbf{C}'(\lambda^*) = 0$. GKE($\lambda$), however, continues to be self-financing even for higher values of λ so long as $\lambda < \overline{\lambda}$, where $\overline{\lambda} \approx 0.87$; this is observable in Figure 4(a). It is also worth pointing out that $\Theta(\lambda^*) \approx 0.6$. Not surprisingly, this number is smaller than $\ln(2) \approx 0.69$, which is the (sufficient) bound given by Theorem 2.3.

D For Online Publication: Analysis of the NDD Proposal

We set up a discrete model to analyze the NDD proposal in this section. We remark that very similar results also hold in a continuum model (i.e. a model similar to the model in Section 2, which would not include a queue explicitly); a discrete model, however, allows us to explicitly model the waiting list as a queue.



(a) The value of $\frac{\mathbf{C}(0)-\mathbf{C}(\lambda)}{\mathbf{C}(0)}$ is plotted.

(b) The blue and the red curve are $f_m(\lambda), \Theta(\lambda)$, respectively (with $\Theta(0) = 1$).

Figure 4: Reduction of healthcare costs in λ.

Briefly, our model is an M/M/1 queue with stochastic abandonments. Suppose we have a queue with Poisson arrival rate of m_d for clients (who represent domestic pairs). Each client has a sojourn time that is drawn (independently) from an exponential distribution with mean 1; the client abandons the queue after her sojourn time is passed. So, similar to our model for GKE, each client has a departure rate 1.

Service is assumed to be a Poisson process with rate λ. In other words, service is determined by a Poisson clock that ticks with rate S, and for each tick, one client is removed from the queue if it is non-empty. Each tick of the clock represents the arrival of a deceased kidney. For simplicity, we assume each arrival corresponds to a successful transplantation.[33] There are no restrictions on the serving process except that it is a greedy process (a transplant is carried out as soon as one is found). In case of finding multiple potential transplants, the tie is broken arbitrarily. The choice of the tiebreaking rule does not effect the analysis, since the sojourn times are exponential.

In Section 3.1, we analyzed the NDD proposal based on the fact that an ϵ increase in the arrival rate of international patients, m_i, decreases the average waiting time of patients in the national queue by ϵ. We noted that this holds under a mild technical assumption: as long as the average length of national queue does not become too small. Here, we rigorously define the assumption and complete the analysis by proving Lemma D.1. This lemma shows that

[33]This assumption is for technical simplicity, and does not affect the quantitative result significantly: The same limit result still holds if we assume that, for instance, each deceased kidney is compatible with each of the patients with probability p, independently, since in the steady state, the queue is with high probability long enough so that each kidney could be transplanted with high probability.

the average length of the queue (which also represents the average waiting time of national patients) is accurately estimated by $m_d - (m_i + k)$. This would prove the promised claim: an ϵ increase in the arrival rate of international patients decreases the average waiting time of patients in the national queue by ϵ.[34]

Before presenting the lemma, we clarify the mild technical assumption that we require. Define $\lambda = m_i + k$. We need to assume that the $\lambda < m_d - {m_d}^{\delta}$ where δ is a positive constant greater than $1/2$. This roughly means, our analysis works so long as the average length of the queue has not dropped below $\sqrt{m_d}$.

Lemma D.1. *Fix a constant $\delta > 1/2$, and suppose we are given a queue with arrival rate m_d and service rate λ. Clients (patients) who are waiting in the queue perish with rate 1. Assume that $\lambda < m_d - {m_d}^{\delta}$. Let $\bar{x}$ denote the expected length of the queue and $x^* = m_d - \lambda$. Then, we have $|\bar{x} - x^*| \leq o(x^*)$.*

Proof. First we model the queue by a 1-dimensional Markov chain $\mathcal{M}$ in the natural way, i.e. we define the state space to be $V(\mathcal{M}) = \{0, 1, 2, \ldots\}$ where state x corresponds to when there are x patients waiting in the queue. The transitions between the spaces would then be defined in the natural way based on the arrival and departure of patients. More precisely, for any $i \in V(\mathcal{M})$ we use l_i, r_i to denote the transition rate from node i to the left and right nodes, respectively. In particular, we would have $l_i = i + \lambda$, $r_i = m_d$.

It is straightforward to verify that $\mathcal{M}$ is positive recurrent and has a stationary distribution π. To simplify the notation in the course of the proof, we use $\mathbb{E}[Z]$ to denote $\mathbb{E}_\pi[Z]$ for any random variable Z.

We start by writing the balance equations, according to which

$$\frac{\pi_{i+1}}{\pi_i} = \frac{r_{i+1}}{l_i}. \tag{D.1}$$

Suppose $i = x^* + y$, for some $y > 0$. (D.1) then implies

$$\frac{\pi_{i+1}}{\pi_i} = \frac{m_d}{x^* + y + \lambda} = \frac{m_d}{m_d + y} = 1 - \frac{y}{m_d + y}. \tag{D.2}$$

[34] As we mentioned earlier, a very similar result holds in a continuum model as well.

For any $m_d \geq 4$ and any $i \in V(\mathcal{M})$, (D.2) implies that

$$\frac{\pi_{i+1}}{\pi_i} \leq 1 - {m_d}^{-1/2}$$
$$\frac{\pi_i}{\pi_{x^*}} \leq 1 - {m_d}^{-1/2}$$

holds when $i - x^* > 2{m_d}^{1/2}$. The above two equations imply that for any i with $i - x^* > 2{m_d}^{1/2}$ we have

$$\begin{aligned}\frac{\pi_i}{\pi_{x^*}} &\leq \left(1 - {m_d}^{-1/2}\right)^{i - x^* - 2{m_d}^{1/2}} \\ &\leq e^{-{m_d}^{-1/2} \cdot (i - x^* - 2{m_d}^{1/2})} \\ &= e^{2 - \frac{i - x^*}{\sqrt{m_d}}} \end{aligned} \tag{D.3}$$

Now, we can establish that $\mathbb{E}_{x\sim\pi}[x] \leq x^* + O(\sqrt{m_d})$. This is done by applying (D.3) as follows

$$\begin{aligned}\bar{x} = \mathbb{E}_{x\sim\pi}[x] &\leq x^* + \sum_{i=0}^{\infty} \pi_{x^* + i\sqrt{m_d}} \cdot (i+1)\sqrt{m_d} \\ &\leq x^* + \sum_{i=0}^{\infty} e^{2 - \frac{x^* + i\sqrt{m_d} - x^*}{\sqrt{m_d}}} \cdot (i+1)\sqrt{m_d} && \text{(D.4)} \\ &= x^* + \sqrt{m_d} \sum_{i=0}^{\infty} e^{2-i} \cdot (i+1) = x^* + O(\sqrt{m_d}) = x^* + o(x^*), && \text{(D.5)}\end{aligned}$$

where (D.4) holds by (D.3) and (D.5) holds because $x^* \geq m_d^{\delta}$ and $\delta > 1/2$.

To complete the proof of the lemma, it remains to show that $\bar{x} \geq x^* - o(x^*)$. The proof for this part is similar to the previous part. Now, suppose $i = x^* - y$, for some $y > 0$. (D.1) then implies

$$\frac{\pi_i}{\pi_{i+1}} = \frac{x^* - y + \lambda}{m_d} = \frac{m_d - y}{m_d} = 1 - \frac{y}{m_d}. \tag{D.6}$$

The above bound, followed by a calculation similar to what derived (D.3) implies

$$\frac{\pi_i}{\pi_{x^*}} \leq e^{2 - \frac{x^* - i}{\sqrt{m_d}}} \tag{D.7}$$

for all $i < x^*$.

Next, we use (D.7) to provide a lower bound for $\bar{x}$:

$$\bar{x} = \mathbb{E}_{x\sim\pi}[x] \geq x^* - \sum_{i=0}^{x^*/\sqrt{m_d}} \pi_{x^*-i\sqrt{m_d}} \cdot (i+1)\sqrt{m_d}$$

$$\geq x^* - \sqrt{m_d} \sum_{i=0}^{x^*/\sqrt{m_d}} \pi_{x_*} \cdot e^{2-i} \cdot (i+1) \tag{D.8}$$

$$= x^* - O(\sqrt{m_d}) = x^* - o(x^*) \tag{D.9}$$

where (D.8) holds by (D.7) and (D.9) holds because $x^* \geq m_d^\delta$ and $\delta > 1/2$.

□

E For Online Publication: Analysis of the NDD Proposal under the Fixed Soujorn Time Assumption

E.1 Model

Briefly, our model is an M/D/1 queue with abandonments. Suppose we have a queue with Poisson arrival rate of C for clients (who represent domestic pairs). Each client waits in the queue for 1 unit of time, and abandons the queue if not serviced by then. This unit of time represents the (currently) 33 month after which the patient's expenses are covered by Medicare.

Service is assumed to be a Poisson process with rate S. In other words, service is determined by a Poisson clock that ticks with rate S, and for each tick, one client is removed from the queue if it is non-empty. Each tick of the clock represents the arrival of a deceased kidney. For simplification, we assume that each arrival corresponds to a successful transplantation.[35] The serving process is a Last-Come-First-Served (LCFS); so, in response to each tick of the clock, the last client who arrived is removed from the queue. The LCFS process is the optimum policy for a cost-minimizing planner whose goal is minimizing the expected total waiting time (in the steady state).

[35]This assumption is made for technical simplicity, and does not affect the quantitative result significantly: The same limit result still holds if we assume that, for instance, each deceased kidney is compatible with each of the patients with probability p, independently. The same limit result holds even under this assumption, since in the steady state, the queue is with high probability long enough so that each kidney could be transplanted with high probability.

Theorem E.1. *For any constant $\delta > 0$, average waiting time of a client is $C - S + O(1)$, as long as $S \leq C(1-\delta)$.*

The proof for theorem E.1 is done in two steps. In the Step 1, we show that the average waiting time is at least $C - S$. In Step 2, we show that the average waiting time is at most $C - S + O(1)$. The implication of Theorem E.1 is that, when C is large and S not too close to C (average queue length is not too short), $C - S$ is an accurate estimate for the average length of the queue.

E.2 Definitions

It helps in our analysis to consider a stochastic process similar to the process that we described, but without abandonments. So, in this alternative process, the clients never leave the queue unless they are served (and everything else would remain identical to the process with abandonments). Let $\mathcal{P}, \mathcal{P}'$ respectively denote the stochastic processes with and without abandonment.

Denote the waiting time of a client c in $\mathcal{P}$ by $\mathsf{w}(c)$; similarly, denote the waiting time of a client c in $\mathcal{P}'$ by $\mathsf{w}'(c)$. We denote by w, w' the average size of the pool in the steady state per unit of time in $\mathcal{P}, \mathcal{P}'$, respectively; i.e.

$$w = C \cdot \mathbb{E}\left[\mathsf{w}(c)\right], \ w' = C \cdot \mathbb{E}\left[\mathsf{w}'(c)\right].$$

Note that in the steady state, the expected size of the pool is also equal to the average (per unit of time) expected waiting time.

E.3 Step 1: The Lower Bound

To establish the lower bound on w, we first analyze the stochastic process $\mathcal{P}'$. Let p denote the *probability of for a client to be served* in $\mathcal{P}'$. In Lemma E.2 we prove that $p = S/C$. Then, in Lemma E.3, we show that any client is served with probability at most p in $\mathcal{P}$. Applying Little's law on the model with abandonments would then imply that $w \geq C(1-p) = C - S$, which concludes Step 1. We finish this step by proving Lemmas E.2 and E.3.

Lemma E.2. $p = S/C$.

Proof. Fix a client c. There are two possible scenarios in which c could be served: (i) A tick from the clock (i.e. server) right after the arrival of c, and (ii) Arrival of a client c' right after the arrival of c. The probability of scenario (i) is $\frac{C}{C+S}$. The probability that c is served in scenario (ii) is $\frac{S}{C+S} \cdot p^2$; it is the probability of arrival of a new client c' times the probability that c' is served, times the probability that c itself is served after that (which is assumed to be p). Thus, we have the following equation:

$$p = \frac{S}{C+S} + \frac{C}{C+S} \cdot p^2,$$

the solution to which is $p = S/C$. □

Lemma E.3. *In the model with abandonments, any client is served with probability at most p.*

Proof. Run the process without abandonments ($\mathcal{P}'$) and couple the process with abandonments ($\mathcal{P}$) with it. The coupling is done in the natural way: For each stochastic event that happens in $\mathcal{P}'$ (arrival of a client or a tick from the clock), let the same event happen in $\mathcal{P}$. We claim that in any sample path, for any client c, the following is true: c is served in $\mathcal{P}$ if it is served in $\mathcal{P}'$. Note that to complete the proof of the lemma, we just need to prove this claim.

We prove a stronger claim instead: a client is served in $\mathcal{P}$ iff it is served in $\mathcal{P}'$ in less than a unit of time. The key to prove this fact is that if , in $\mathcal{P}'$, a client c is served after waiting one unit of time, it means that in $\mathcal{P}$, all the unserved clients who arrived before c and c itself have abandoned (this is due to the LCFS policy and the fact that all clients wait for one unit of time). On the other hand, if, in $\mathcal{P}'$, a client c is served in less than a unit of time, it means that c is also served in $\mathcal{P}$. Moreover, it means that all the clients who arrived after c in $\mathcal{P}'$ are served in less than a unit of time, which also implies all the clients who arrived after c in $\mathcal{P}$ are served in less than a unit of time. Exactly the same scenario must happen when a client c is served in $\mathcal{P}$, which means c is served in $\mathcal{P}$ iff it is served in $\mathcal{P}'$ in less than a unit of time. □

E.4 Step 2: The Upper Bound

Consider the process without abandonments, $\mathcal{P}'$, and let $f : [0, \infty] \to \mathbb{R}_+$ denote the PDF of service time in $\mathcal{P}'$ (Note that since $p < 1$, we have $f(\infty) > 0$). We provide an upper bound on w in terms of f. In the model with abandonments, we have:

$$w/C \leq \int_0^1 t f(t) \,\mathrm{d}\, t + \int_{t \in [1,\infty)} f(t) \,\mathrm{d}\, t + (1-p) \cdot 1. \tag{E.1}$$

To see why the above inequality holds, we interpret its RHS considering the corresponding coupled process without abandonment ($\mathcal{P}'$). The first summand accounts for the waiting time of clients c who are served in $\mathcal{P}$; we have corresponded it to the clients in $\mathcal{P}'$ who have waiting time less than 1. The second summand accounts for the waiting time of a subset of unserved clients in $\mathcal{P}$, which are corresponded to the clients c in $\mathcal{P}'$ with $1 < \mathsf{w}'(c) < \infty$; note that such clients only wait for 1 unit of time in $\mathcal{P}$, and thus rather than multiplying the term inside the integral by t, we have multiplied it by 1. The third summand is an upper bound on the waiting time of the rest of the unserved clients in $\mathcal{P}$, which are corresponded to the clients c in $\mathcal{P}'$ with $\mathsf{w}'(c) = \infty$; these clients also wait only for 1 unit of time in $\mathcal{P}$.

Next, we write a simple upper bound for the RHS of (E.1) and rewrite (E.1) as follows:

$$\begin{aligned} w/C &\leq \int_{t\in[0,\infty)} t f(t)\,\mathrm{d}\,t + (1-p)\cdot 1 \\ &= p\cdot \mathbb{E}\left[\mathsf{w}'(c)\middle|\mathsf{w}'(c)<\infty\right] + (1-p)\cdot 1. \end{aligned} \tag{E.2}$$

In writing the above equality, we have used the fact that $\int_{t\in[0,\infty)} t f(t)\,\mathrm{d}\,t = p\cdot\mathbb{E}\left[\mathsf{w}'(c)\middle|\mathsf{w}'(c)<\infty\right]$, for any client c.

Next, we show that $\mathbb{E}\left[\mathsf{w}'(c)\middle|\mathsf{w}'(c)<\infty\right] \leq \frac{\theta_\delta}{C+S}$, for some constant $\theta_\delta > 0$ (as we will see later, θ_δ is independent of C, S, but could depend on δ). If we prove this, then (E.2) implies that

$$w \leq \frac{p\theta_\delta\cdot C}{C+S} + C(1-p) = \frac{\theta_\delta S}{C+S} + C - S \leq \theta_\delta/2 + C - S,$$

which would prove the claim. Thus, the proof is finished by proving the following lemma.

Lemma E.4. $\mathbb{E}\left[\mathsf{w}'(c)\middle|\mathsf{w}'(c)<\infty\right] \leq \frac{\theta_\delta}{C+S}$.

Proof. Consider a random walk on integers that starts at origin. At each step, the walk goes either to left or right, with probabilities $P_l = \frac{S}{C+S}$ and $P_r = \frac{C}{C+S}$, respectively. We use this random walk to prove the lemma. Let q be the probability that the random walk returns to the origin conditioned on its first step being to the right. Then,

$$\mathbb{E}\left[\mathsf{w}'(c)\middle|\mathsf{w}'(c)<\infty\right] = P_r\cdot q\cdot\frac{1}{C+S}, \tag{E.3}$$

where in the RHS of (E.3), the first factor is the probability that the first step is to the right (which corresponds to the arrival of another client c' right after the arrival of c), the second factor is the probability of returning to the origin (which corresponds to the probability of serving c conditioned on the arrival of c'), and the third factor accounts for the speed of movements in the random walk (the rate of movement in the random walk is $C+S$). Consequently, the RHS of (E.3) accounts for $\mathbb{E}\left[\mathsf{w}'(c)\middle|\mathsf{w}'(c)<\infty\right]$. To complete the proof, we will show that q is in fact a constant independent of C, S (but possibly dependent to δ); setting $\theta_\delta = qP_r$ would then conclude the proof.

To compute q, we count the number of sample paths of length n which start with going to right and return to origin after n steps (n being even). Denote the number of such paths by n, and see that

$$q = \frac{1}{P_r} \cdot \sum_{n=2}^{\infty} (P_l P_r)^{n/2} C_n.$$

It is a well-known fact that the number of such paths, C_n, is given by

$$C_n = \frac{1}{n/2} \cdot \binom{n-2}{n/2-1}.$$

Using this fact, and the fact that $S \leq C(1-\delta)$, we can write:

$$\begin{aligned}
q &= \frac{1}{P_r} \cdot \sum_{n\in\mathbb{N}, n \bmod 2=0}^{\infty} \left(\frac{CS}{(C+S)^2}\right)^{n/2} \cdot \frac{1}{n/2} \cdot \binom{n-2}{n/2-1} \\
&\leq \frac{1}{P_r} \cdot \sum_{n\in\mathbb{N}, n \bmod 2=0}^{\infty} \left(\frac{1-\delta}{(2-\delta)^2}\right)^{n/2} \cdot \frac{1}{n/2} \cdot \binom{n-2}{n/2-1} \\
&\leq \frac{1}{P_r} \cdot \sum_{n\in\mathbb{N}, n \bmod 2=0}^{\infty} \frac{1}{n/2} \cdot {\beta_\delta}^{n/2} \cdot 4^{n/2-1} \\
&= \frac{1}{2P_r} \cdot \sum_{n\in\mathbb{N}, n \bmod 2=0}^{\infty} \frac{1}{n} \cdot (4\beta_\delta)^{n/2},
\end{aligned} \tag{E.4}$$

where β_δ denotes $\frac{1-\delta}{(2-\delta)^2}$. Now, see that (E.4) is a constant since $\beta_\delta < 4$. To finish the proof, we rewrite (E.3) by plugging the upper bound (E.4) into it to get

$$\mathbb{E}\left[\mathsf{w}'(c)\middle|\mathsf{w}'(c)<\infty\right] \leq \frac{\theta_\delta}{(C+S)}, \tag{E.5}$$

where we have denoted the constant $\frac{1}{2} \cdot \sum_{n\in\mathbb{N}, n \bmod 2=0}^{\infty} \frac{1}{n} \cdot (4\beta_\delta)^{n/2}$ by θ_δ. $\square$

As we mentioned earlier, Lemma E.4 and (E.2) together imply that

$$w \leq \frac{p\theta_\delta \cdot C}{C+S} + C(1-p) = \frac{\theta_\delta S}{C+S} + C - S \leq \theta_\delta/2 + C - S,$$

which concludes Step 2 by establishing the desired upper bound on w.

F For Online Publication: Computational Experiments

In this section we complete the comparison of the continuum model and its counterpart discrete model by providing the simulation results for the discrete model in Subsection F.1. We also compare our closed-form solutions to the discrete model (derived in our addendum) with the values obtained from our simulations and observe that when the large market assumption is met, the closed-form solutions provide a reasonable approximation.

Simulation instances. We ran our simulations on two instances of the market. Each instance is characterized by parameters m, d, θ, which are interpreted as follows. Domestic and international pairs arrive according to independent poisson processes with rates $m, \lambda m$. Any two pairs (domestic or international) are compatible with probability $p = d/m$, independently. (One can interpret d as the average degree in the domestic pool if no matches are ever made.) Given these parameters, we plot several characteristics of the market for λ varying from 0 to 1 with increments of 0.05. In both instances we assume $\theta = 1$, i.e. the average cost of dialysis per person per life is equal to the surgery cost per person. This is not a crucial assumption and we observe the same results in our simulations for any $\theta > \ln 2$ (see Theorem 2.2).

Closed-form solutions. In our addendum, we show that the expected size of the domestic and international pools in the steady-state are approximately $\ln\left(\frac{2}{1+\lambda}\right)\frac{m}{d}$ and $\ln\left(\frac{1}{1-\lambda}\right)\frac{m}{d}$, respectively, where our approximation notion suppresses lower order terms that relatively vanish as the term m/d goes to infinity. Roughly speaking, our closed-form solutions work well when m is sufficiently large, and d is sufficiently small relative to m. In what follows we compare our closed-form solutions with the values obtained from simulating the stochastic process and observe that our closed-form solution becomes more accurate as m/d grows.

F.1 A simulation with $m = 1000, d = 10$

Consider a market with $m = 1000$ and $d = 10$. Note that having $d = 10$ does not mean that the average degree of each arriving node is 10; rather, the average degree would be 10 if no matches are made at all. When matches are formed greedily, as we will see in the simulations, the domestic pool has an average size close to $\ln\left(\frac{2}{1+\lambda}\right)\cdot\frac{m}{d}$, which means the average degree of each arriving node in the domestic pool would be about $\ln\left(\frac{2}{1+\lambda}\right)$.

We run our simulation in 21 scenarios with λ varying from 0 to 1 with increments of 0.05. In each scenario, we simulated the system for 10,000 events, i.e. 10,000 arrivals or departures. Each scenario takes between 150 to 300 rounds. This period is long enough to observe convergence in the average of the random variables that we study.

Average (domestic) pool size. Figure 5 plots the average pool size observed in simulations as well as the average pool size that we predict theoretically.

Dialysis and surgery costs. Figure 6 plots the surgery and dialysis costs. This figure is suggesting that the average dialysis cost is a convex function of λ, which is also confirmed by our closed-form estimates. On the other hand, the surgery cost is almost a linear function of λ. Putting these two facts together shows the existence of a threshold λ^* below which the GKE proposal is self-financing. This is clarified further in Figure 7, where the difference between the reduction in the dialysis cost and the surgery cost is plotted as a function of λ. We used the data of this figure in Figure 3 as well, where it was compared the analytical solution to the continuum model.

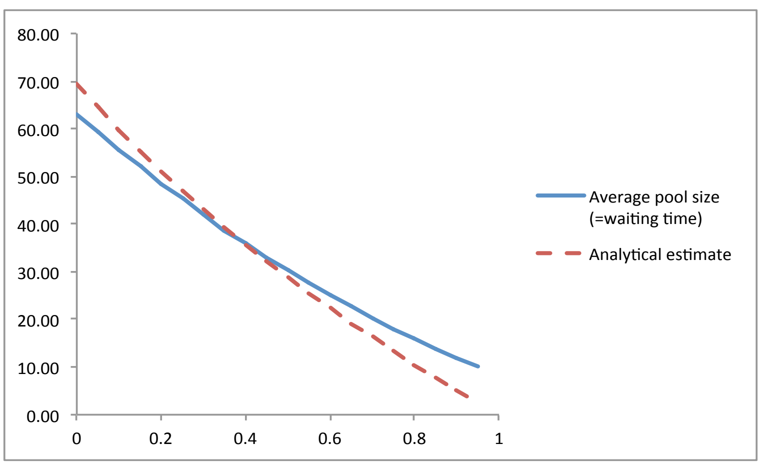


Figure 5: The observed average pool size compared to the analytical estimate $\ln\left(\frac{1+\lambda}{2}\right)\cdot\frac{m}{d}$.

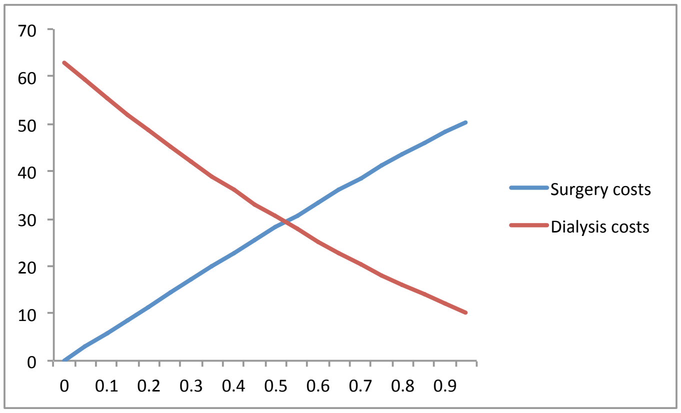


Figure 6: Convexity of the dialysis cost and linearity of the international patients' transplant costs.

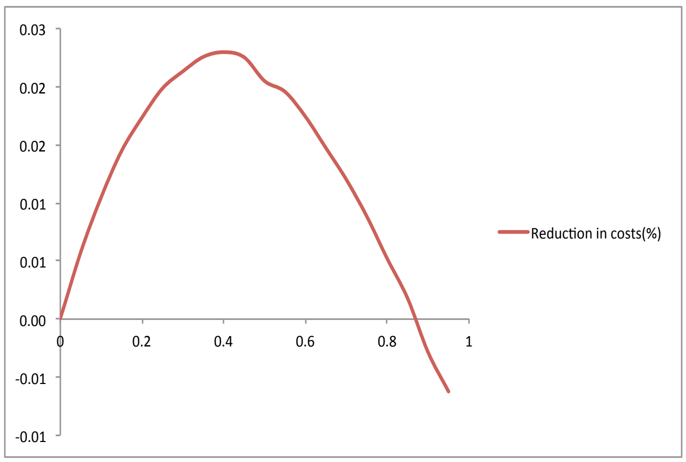


Figure 7: The amount of reduction in the healthcare costs.

F.2 A simulation with $m = 10{,}000\,, d = 50$

Next, we consider a market with $m = 10000$ and $d = 50$. Figure 8 plots the average pool size observed in simulations as well as the average pool size that we predict theoretically. We observe that our closed-form approximations are more accurate here as m is larger and d is smaller relative to m. Figure 9 plots the surgery and dialysis costs. Figure 10 plots the percentile of reduction in total healthcare costs, i.e. $\frac{\mathbf{C}(0)-\mathbf{C}(\lambda)}{\mathbf{C}(0)}$.

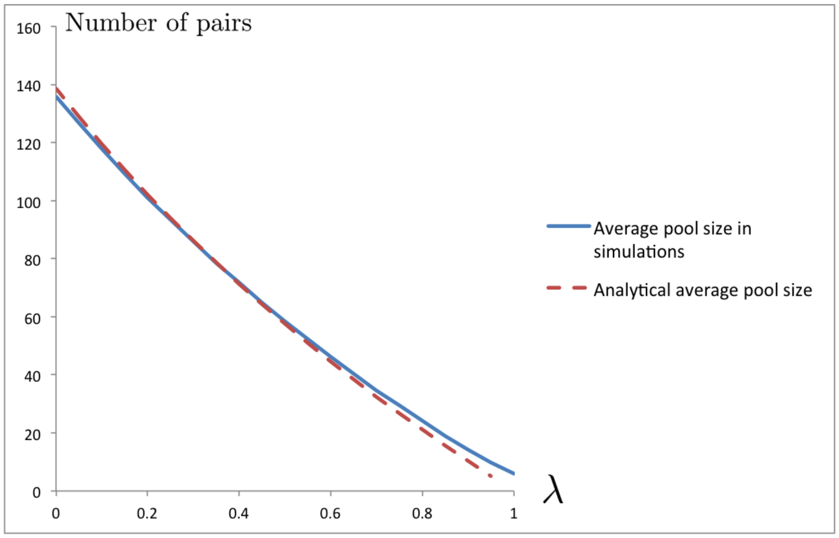


Figure 8: The observed average pool size is very close to our analytical estimate $\ln\left(\frac{1+\lambda}{2}\right) \cdot \frac{m}{d}$.

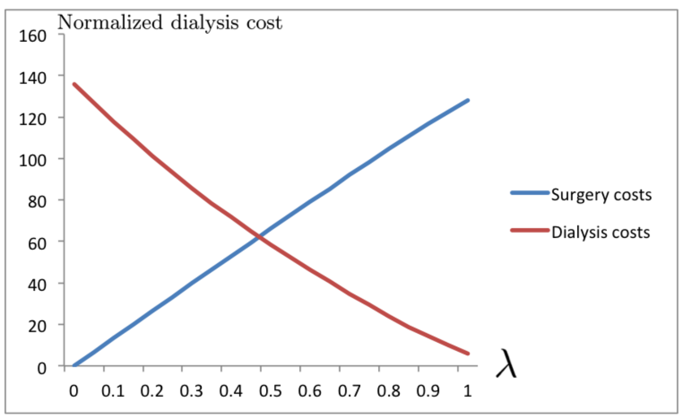


Figure 9: The dialysis cost and international patients' transplant cost are respectively a convex and (almost) linear function of λ. The measure on the vertical axis is *normalized dialysis cost*, i.e. dialysis cost per patient per time unit.

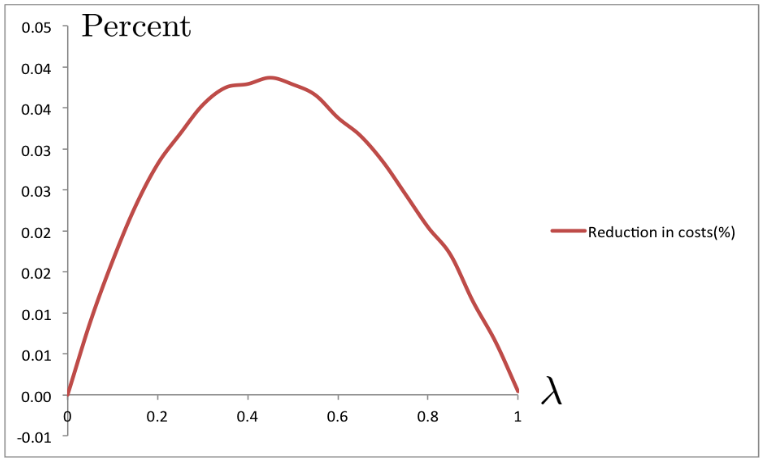


Figure 10: The percentage of reduction in healthcare costs.